NUMERICAL OPTIMIZATION FOR INTERNAL EXPANDING BRAKE

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THESIS

NUMERICAL OPTIMIZATION FOR
INTERNAL EXPANDING BRAKE

by

MORDECHAI PEER

March 1981

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This report deals with design optimization of Internal-Expanding Rim Brakes. A computer program was developed to calculate the actuating force, torque, stopping time and drum temperature. The drum temperature is calculated by the finite difference method. A comparison of results has been made using a simplified equation that is in common use in engineering texts. Numerical optimization is shown to be a convenient tool for brake design.

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Numerical Optimization for Internal Expanding Brake

by

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ABSTRACT

This report deals with design optimization of Internal-Expanding Rim Brakes. A computer program was developed to calculate the actuating force, torque, stopping time and drum temperature. The drum temperature is calculated by the finite difference method.

A comparison of results has been made using a simplified equation that is in common use in engineering texts.

Numerical optimization is shown to be a convenient tool for brake design.

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SYMBOLS AND ABBREVIATIONS

A. ENGLISH LETTER SYMBOLS

a	Distance from pivot to the center of rotation (m).
A	Area of one lining shoe (m^2).
b	Width of friction material (m).
B_i	Biot modulus.
c	Specific heat (J/Kg-^{0}C).
C	Thermal capacity (J/^{0}C).
d	Distance from actuating force to the hinged pin (m).
dc	Rate of deceleration (m/sec^2).
E	Kinetic energy (J).
f	Frictional force (N)
F	Actuating force (N).
F_0	Fourier modulus.
g	Gravity constant (m/sec^2).
h	Convection heat transfer coefficient (W/m^2 ^{0}-C).
k	Thermal conductivity (W/m-^{0}C).
M_f	Friction moment (N-m).
M_n	Normal moment (N-m).
N	Normal force (N).
p	Pressure between lining and drum at any point (N/m^2).
p_a	Maximum pressure between lining and drum (N/m^2).
Q	Heat generated (W).
r	Inside drum radius (m).
R	Wheel radius (m).
R_{th}	Thermal resistance (^{0}C/W).
t	Time (sec.)
tk	Thickness (m).
T	Temperature (^{0}C).
T_0	Torque (N-m).
V	Velocity (m/sec.).
V_0	Volume (m^3).
W	Vehicle weight (N).

B. NOTATION

R_{ij} The thermal resistance between node i and the adjoining node j.

T_i^p The temperature of node i at time step p.

C. GREEK LETTER SYMBOLS

θ The angle between the hinged pin and an element area on the lining.

θ_a The angle at which the pressure between the lining and drum is maximum.

μ Friction coefficient.

μ_c Cold friction coefficient.

μ_h Hot friction coefficient.

α Thermal diffusivity (m^2/sec.).

Δ Finite increment.

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I. INTRODUCTION

Brakes are mechanical devices for retarding the motion of a vehicle or machine by means of friction. Because of the similarity of their functions, many clutches may also be included here, assuming centrifugal forces are accounted for.

A simplified dynamic representation of a brake is shown in Fig. 1. Two masses with inertias, I_1 and I_2, rotating at the respective angular velocities ω_1 and ω_2 (one of which may be zero), are to be brought to the same speed by engaging the brake.

The friction brake has three basic elements; two opposing friction surfaces and a mechanism for forcing the friction surfaces into contact. Whenever a friction brake is engaged to join two members having relative motion, there is a period of slip which may last several seconds. This slip is one of the chief merits of the friction brake; it absorbs shocks and prevents excessive torsional stresses on the power transmission system. On the other hand, slip is the limiting factor in friction clutch and brake performance; for heat is generated in proportion to slip, torque transmitted, and period of slip.

The following parameters are of interest in analyzing the performance of these devices;

1. The actuating force.
2. The torque transmitted.
3. The temperature rise.
4. The slip time.

This report deals with Internal-Expanding Rim Brakes. This formulation also applies to internal-expanding clutches if centrifugal forces are accounted for.

II. INTERNAL-EXPANDING RIM CLUTCHES AND BRAKES

A. GENERAL MACHANICAL PRINCIPALS

A brake or clutch assembly, uses a brake shoe to which is attached a friction material, called lining. The lining is riveted or bonded to the brake shoe as shown in Fig. 2. The brake shoe is pivoted at a fixed point and the other end is subjected to a force which presses the shoe in contact with the drum. The force between the brake and the drum is radial as the drum rotates. If a point on the rotating drum surface first makes contact with the shoe at the end nearest the pivot, the shoe is termed a "trailing shoe". If it first makes contact at the other end the shoe is termed "leading shoe", the latter giving a higher braking torque than the former for a given braking force.

The friction between the lining and the drum creates heat which is basically the conversion of energy of motion of the vehicle or machine to thermal energy at the friction surfaces, namely the lining and the drum. This heat is then dissipated and absorbed by the drum by conduction, convection and radiation into the atmosphere.

B. FRICTION FUNDAMENTALS AND MATERIALS

Friction mechanisms, such as brakes, are systems for converting mechanical energy into heat. Several basic factors affect friction and wear of materials used in brake systems. The main factors are temperature, pressure, speed, surface roughness, and type of material. Some organic or molded friction materials show no change in friction characteristics with pressure, while others such as sintered-metal materials decrease in friction coefficient as pressure is increased. For metallic friction materials there is also a decrease in coefficient of friction as speed

increases. Temperature effects upon the coefficient of friction vary widely with the type of materials used.

In a two-shoe internal expanding brake there is a tendency for the brake drum to deform under hard application. Drums become elliptical and the force to do this is quite high and contributes to friction force.

A brake or clutch friction material should have the following characteristics to a degree which is dependent upon the severity of the service:

1. A high and uniform coefficient of friction.
2. The ability to withstand high temperatures, together with good heat conductivity.
3. Properties which are not affected by environmental conditions such as moisture.
4. Good resiliency.
5. High resistance to wear, scoring and galling.

C. BRAKE DRUMS

One of the primary functions of a brake drum is that of absorbing and dissipating the heat developed during the application of the brake. A brake drum is a heat sink into which heat goes after it is created by the rubbing friction of the brake lining contact to drum. The brake shoe and lining permanently fixed on the axle, when actuated, contacts the drum under pressure to cause the friction to stop the vehicle. The energy of motion of a vehicle is converted to thermal energy by the brake assemblies. A brake drum must have the capacity to absorb and dissipate this heat energy within the limits of the brake heat input. If this is not the case, the drum expands and the brakes fade or fail. The greater the mass of the drum, the more heat it can absorb and store until such time as the heat can be dissipated by convection and radiation [Ref. 1].

An ideal brake drum would have the following characteristics;

1. High structural strength to resist bursting forces.
2. Uniform coefficient of friction.
3. Hard surface to resist scoring.
4. High heat conductivity to rapidly conduct heat away from braking surfaces.
5. High emissivity factor to radiate heat from the drum surface to the atmosphere.
6. High heat storage capacity to store heat from successive brake applications until it can be dissipated.
7. Good machinability to permit boring of the drum.

D. STATIC AND DYNAMIC ANALYSIS

1. Assumptions

In developing the equations, the following assumptions have been made;

a. The pressure at any point on the shoe is proportional to the moment arm of this point from the pivot.
b. The effect of centrifugal force may be neglected.
c. The shoe is assumed to be rigid.
d. The friction coefficient is a linear function of temperature and it does not vary with pressure, wear and environment.

2. Pressure Concept

To analyze an internal shoe refer to Fig. 2, which shows a shoe pivoted at a fixed point with the actuating force acting at the other end of the shoe. The mechanical arrangement does not permit pressure to be applied at the pivot, therefore the pressure at this point is zero. If the shoe rotates through a small angle about A, the radial movement of any point on the arc of contact, is proportional to the moment arm of this point from the pivot. Assuming that the material of the brake lining and support obey Hooke's law, the pressure at this point will also be proportional to this moment arm. The distance is

proportional to $\sin\theta$. Therefore, the relations between pressure at any point and the maximum pressure, p_a, will be given by the following formula;

$$\frac{p}{\sin\theta} = \frac{p_a}{\sin\theta_a} \qquad (1)$$

From this formula it can be seen that the frictional material at the heel, contributes very little to the braking action, therefore it is better to begin the friction material at an angle θ_1 greater than, say 0.15 rad. It can be seen also that the pressure will be maximum when $\theta = 90^0$ or if the toe angle θ_2 is less than 90^0, then the pressure will be maximum at the toe. For good performance it is recommended to concentrate as much frictional material as possible in the neighborhood of the point of maximum pressure [Ref. 2].

3. Actuating Force and Torque Calculation

From Fig. 2, it can be seen that the differential normal force on an element area of the lining will be;

$$dN = pdA \qquad (2)$$

where dA is an area element of the lining and it's magnitude is;

$$dA = rbd\theta \qquad (3)$$

In Equation 3, r is the inside drum radius and b is the drum width. Substituting for p and dA gives;

$$dN = \frac{p_a b r \sin\theta}{\sin\theta_a} d\theta \qquad (4)$$

At the same point the differential frictional force is;

$$df = \mu dN \qquad (5)$$

where μ is the coefficient of friction.

The actuating force, F, can be calculated using the fact that the summation of the moments about the hinge pin is zero. The moment due to frictional forces is;

$$M_f = \int_{\theta_1}^{\theta_2} (r-\cos\theta)df \qquad (6)$$

where a is the distance from the pivot to the center of rotation. Substituting the value of dF and integrating from θ_1 to θ_2 gives;

$$M_f = \frac{\mu p_a b r^2}{\sin\theta_a}\{(\cos\theta_1-\cos\theta_2)+\frac{a}{2r}(\sin^2\theta_1-\sin^2\theta_2)\} \qquad (7)$$

where μ is assumed to be constant along the lining. Similarly the moment due to normal forces is given by;

$$M_n = \int_{\theta_1}^{\theta_2} a\sin\theta dN \qquad (8)$$

Substituting the value of dN and integrating from θ_1 to θ_2 gives;

$$M_n = \frac{p_a bra}{\sin\theta_a}\{0.5(\theta_2-\theta_1)-0.25(\sin\theta_2-\sin\theta_1) \qquad (9)$$

The actuating force must balance the moments, therefore;

$$F = \frac{M_n-M_f}{d} \qquad (10)$$

where d is the distance from the hinge to the point of application of F. The torque applied to the drum by the brake shoe is;

$$T_0 = \int_{\theta_1}^{\theta_2} rdf \qquad (11)$$

After substituting the value of df and integrating ;

$$T_0 = \frac{\mu p_a b r^2}{\sin\theta_a}(\cos\theta_1-\cos\theta_2) \qquad (12)$$

4. Rate of Heat Generated and Deceleration Calculation

The differential rate of heat generated by an element area of the lining is equal to the velocity of the inside surface of the drum relative to the lining, times the differential frictional force acting on the element area;

$$dQ = \bar{V}_r df \qquad (13)$$

Assuming the brake is on a vehicle wheel with a radius of R, the inside surface velocity is equal to;

$$V_r = \frac{r}{R}V \qquad (14)$$

where V is the velocity of the vehicle and is a function of time.

If $V=V(t)$ then $V_r=V_r(t)$ and the heat generated will be also a function of time. Substituting the values of V_r and df and integrating from θ_1 to θ_2, we get the following formula for the heat generated at any time t,

$$Q(t) = \frac{p_a b\mu}{\sin\theta_a}\left(\frac{r^2}{R}\right)(\cos\theta_1-\cos\theta_2)V(t) \qquad (15)$$

The kinetic energy of a vehicle of weight W is given by;

$$E = \frac{1}{2}\left(\frac{W}{g}\right)V^2 \qquad (16)$$

Note that if the brake is on a four wheel vehicle, there will be eight shoes. Assuming all are leading shoes, each will stop one-eight of the vehicle weight, so W/8 must be used in Equation (16). The rate of change in the kinetic energy is;

$$\frac{dE}{dt} = \left(\frac{W}{g}\right)V\frac{dV}{dt} \qquad (17)$$

From the energy conservation law the rate of change in the kinetic energy is equal to the heat generated;

$$Q(t) = \frac{dE}{dt} \tag{18}$$

Substituting the value of Q(t) and dE/dt, it is seen that the velocity V(t) cancels and so the deceleration is not a function of time. Therefore the deceleration, dc, is;

$$dc = \frac{dV}{dt} = \left(\frac{g}{W}\right)\frac{p_a b\mu}{\sin\theta_a}\left(\frac{r^2}{R}\right)(\cos\theta_1-\cos\theta_2) \tag{19}$$

The velocity at any time is;

$$V = V_i - dct \tag{20}$$

where V_i is the initial velocity. Substituting the velocity in Equation (16), yields the rate of heat generated as a function of time,

$$Q(t) = \frac{p_a b\mu}{\sin\theta_a}\left(\frac{r^2}{R}\right)(\cos\theta_1-\cos\theta_2)(V_i-dct) \tag{21}$$

In this study the friction coefficient was taken as constant up to a temperature of 90 °C and after 90 °C, decreases linearly to zero at a specified temperature, T_{max}:

$$\mu = \begin{cases} \mu_c & T \le 90\ ^0C \\ \mu_c - \dfrac{\mu_c-\mu_h}{\Delta T}(T-90) & 90\ ^0C < T \le T_{max.} \\ 0 & T > T_{max.} \end{cases} \tag{22}$$

where μ_c is the cold coefficient of friction and μ_h is the hot coefficient of friction.

E. SURFACE TEMPERATURE CALCULATION

Since the function of a brake is to convert kinetic energy into heat, surface temperatures of brake linings and drums are most important. Therefore it is necessary to know the temperature of the mechanism during and after any stop. The temperatures were calculated by the finite difference method.

1. Assumptions

a. One dimensional heat flow-The heat flow is from the inner surface to the outer surface of the drum.

b. Constant heat transfer coefficient.

c. No heat dissipated by radiation.

d. The heat is generated on the inner surface.

2. Temperature Analysis

a. Theory

The differential equation to be solved in order to find the temperature in the drum, based on the assumptions, is;

$$\frac{\partial^2 T}{\partial x^2} + \frac{Q}{k} = \left(\frac{1}{\alpha}\right)\frac{\partial T}{\partial t} \tag{23}$$

with the following boundary conditions:

at x=0 heat is generated,

at x=tk heat is transfered to the atmosphere by convection.

In the equation above k is the thermal conductivity, α is the thermal diffusivity, t is time and tk is the drum thickness. This equation can be solved by the finite difference method [Ref. 3]. The finite difference model used here is shown in Fig. 3. The rate of change with time of the internal energy of a node i is approximated by;

$$\frac{\Delta E}{\Delta t} = \rho c \Delta V_0 \frac{T_i^{p+1} - T_i^p}{\Delta t} \tag{24}$$

where ρ is the density, c is the specific heat and V_0 is the drum volume.

Now define the thermal capacity as

$$C_i = \rho_i c_i \Delta V_{0_i} \tag{25}$$

The forward difference equation for all nodes and boundary conditions is;

$$Q_i^p+\sum\frac{T_j^p-T_i^p}{R_{th,ij}} = C_i\frac{T_i^{p+1}-T_i^p}{\Delta t} \tag{26}$$

where $R_{th,ij}$ is the thermal resistance

Solving the above equation for T_i^{p+1} gives;

$$T_i^{p+1} = (Q_i^p+\sum\frac{T_j^p}{R_{th,ij}})\frac{\Delta t}{C_i}+(1-\frac{\Delta t}{C_i}\sum\frac{1}{R_{th,ij}})T_i^p \tag{27}$$

The thermal resistance can be calculated from the geometry and boundary conditions [Ref. 3]. To ensure stability Δt must be equal or less than the following nodal relation;

$$\Delta t < (\frac{C_i}{\sum\frac{1}{R_{th,ij}}}) \tag{28}$$

With the assumptions made, the drum can be viewed as an infinite plate, with heat generated at the surface of the first node, as shown in Fig. 3. It is assumed that in every drum, there are two shoes and that both are leading shoes. Therefore, two times Q_i^p must be taken.

$$T_i^{p+1} = (2Q_i^p+\sum\frac{T_j^p}{R_{th,ij}})\frac{\Delta t}{C_i} + (1- \frac{\Delta t}{C_i}\sum\frac{1}{R_{th,ij}})T_i^p \tag{29}$$

b. Formulation

In the computer program 5 nodes were taken. In order to check accuracy, the program was run with 7 and 10 nodes. In each case the result was the same within 5 ^{0}C. The heat is generated in the inner drum surface. Therefore Q appears in the formula of temperature in the first node and for all the other nodes Q is equal zero. With the assumptions mentioned above, the heat transfer through the drum is solved as a heat transfer problem through an infinite plate, with heat generation at the inner surface and with a heat convection boundary on the outer surface as shown in Fig. 3. Equation (29) can be simplified using two dimensionless parameters, Biot and Fourier modulii,

$$B_i = \frac{h\Delta x}{k} \tag{30}$$

$$F_0 = \frac{\alpha\Delta t}{(\Delta x)^2} \tag{31}$$

The final equations for calculating the temperatures at the nodes now become;

For the first node;

$$T_1^{p+1} = \frac{2Q_1^p\Delta t}{C_1} + (1-2F_0)T_1^p + 2F_0T_2^p \tag{32}$$

For the interior nodes;

$$T_i^{p+1} = F_0\{T_{i-1}^p + T_{i+1}^p + (\frac{1}{F_0} - 2)T_i^p\} \tag{33}$$

For the last node;

$$T_n^{p+1} = 2F_0\{T_{n-1}^p + B_iT_\infty + (\frac{1}{2F_0} - B_i - 1)T_n^p\} \tag{34}$$

F. BRAKE DUTY CYCLE

In addition to the parameters mentioned above the design of a brake depends on the initial speed, final speed, number of stops, and the rest time between each stop. In this analysis a general duty cycle was considered so that the initial speed, final speed and the acceleration period between stops can be different for each part of the design.

In the design examples presented here, a vehicle was stopped four consecutive times with the following cycle;

	Initial Speed m/sec.	Final Speed m/sec.	Rest sec.
1	25.0	0.0	20.0
2	25.0	0.0	20.0
3	25.0	0.0	20.0
4	25.0	0.0	-

III. OPTIMIZATION

A. INTRODUCTION

Engineering analysis using the digital computer has become commonplace. It is less common to use the computer to make the actual design decisions, such as sizing of structural members or placement of mechanical linkages. This may be largely attributed to the fact that fully automated design requires techniques that are unfamiliar to much of the engineering community.

In many engineering problems, it is necessary to determine the minimum or maximum of a function of several variables, limited by various linear and nonlinear inequality constraints. It is seldom possible, in practical applications, to solve these problems directly, and iterative methods are used to obtain the numerical solution. Machine calculation of this solution is, of course, desirable. The CONMIN program is available to solve a wide variety of such problems [Ref. 4].

CONMIN is a FORTRAN program, in subroutine form, for the minimization of a multi-variable function subject to a set of inequality constraints. The basic optimization algorithm is the Method of Feasible Directions [Ref. 5]. The user must provide a main calling program and an external routine to evaluate the objective and constraint functions and to provide gradient information. If analytic gradients of the objective or constraint functions are not available, this information is calculated by finite difference. While the program is intended primarily for efficient solution of constrained problems, unconstrained function minimization problems may also be solved, and the Conjugate Direction Method of Fletcher and Reeves is used for this purpose [Ref. 6].

B. DEFINITION OF TERMS

Most disciplines have a unique set of nomenclature used to describe the concepts within that discipline. Some of the commonly used terms in numerical optimization are summarized here.

Objective- The value of the function which is to be minimized or maximized during the optimization process. Synonyms are cost, merit and payoff. The common mathematical designation is $F(\bar{X})$. In the present study the objective was to minimize the material in the brake drum.

Design variables- The parameters to be changed during the optimization process in order to minimize or maximize the value of the objective function. Synonym; decision variables. The common mathematical designation is the vector $\bar{X}$. Design variables considered in this study include, drum thickness, width, the angle between the hinged pin and the end of the lining, and the distance from the pivot to the center of rotation.

Inequality constraints- One-sided conditions which must be mathematically satisfied for the design to be acceptable. The common mathematical term is $G(\bar{X})<0$ or $G(\bar{X})>0$. If the ineqality condition is satisfied on $G(\bar{X})$, the design is acceptable, (feasible). If it is not satisfied, the design is not acceptable (infeasible). Constraints considered here include, vehicle stopping time, maximum drum temperature, and actuating force.

Side constraints- Upper and lower bounds on the individual design variables $\bar{X}$. The common mathematical representation is $X_i^l < X_i < X_i^u$.

Design space- The n-dimensional mathematical space spanned by the vector of design variables $\bar{X}$.

Active constraint- Constraint $G_j(\bar{X})$ is called active if its value is zero (or near zero for computational purposes).

Inactive constraint- Constraint $G_j(\bar{X})$ is inactive if $G_j(\bar{X})<0$.

Violated constraint- Constraint $G_j(\bar{X})$ is violated if $G_j(\bar{X})>0$.

C. THE OPTIMIZATION PROCESS

The general design optimization problem can be stated mathematically as follows: Find the set of variables X_i, $i=1,2....n$, which will

$$\text{Minimize } F(\bar{X}) \tag{35}$$

Subject to:

$$G_j(\bar{X}) \leq 0 \qquad j=1,2.....m \tag{36}$$

$$X_i^l \leq X_i \leq X_i^u \qquad i=1,2.....n \tag{37}$$

Vector $\bar{X}$ contains the set of independent design variables X_i, $i=1,2....n$. $\bar{X}$ may represent, for example width, thickness, and angles in the brake optimization. The objective function used here is the drum volume.

Equation (36) defines the ineqality constraints imposed on the design. For example, if the temperature on the inner drum surface must not exceed a specified value $\bar{T}$, the associeted design constraint becomes, in normalized form

$$\frac{T_i}{\bar{T}_i} - 1 \leq 0 \tag{38}$$

The lower and upper bounds on the design variables, given by Eq. (37) , limit the region over which the functions $F(\bar{X})$, and $G(\bar{X})$ are defined. These constraints are often referred to as side constraints because they form the sides or bounds of the n-dimensional spaced spanned by the design variables $\bar{X}$.

If all the inequalities of Eqns. (36) and (37) are satisfied, the design is said to be feasible; if any of these conditions are not satisfied, the design is not

feasible. If $F(\bar{X})$ is a minimum and the design is feasible, it is also optimum, or at least, a relative optimum. Note that because the objective and constraints may be nonlinear, there may be multiple minima in the design space that cannot be identified using current methods. While this is a matter for concern, since it is desired to find the true optimum, it must be remembered that the same mathematical conditions exist if the design process is not automated. However, using optimization techniques, it is a simple matter to restart the optimization from several initial points in the design space and thereby improve the probability of obtaining the true optimum design, a process that would be quite time-consuming in manual design.

Equations (35)-(37) define the nonlinear constrained optimization problem. If Eqs.(36) and (37) are not imposed on the design, the optimization problem is defined by Eq.(35) alone and is therefore an unconstrained minimization problem.

Most nonlinear optimization algorithms update the vector of design variables by the iterative relationship;

$$\bar{X}^q = \bar{X}^{q-1} + \alpha\bar{S}^q \tag{39}$$

where q is the iteration number, vector $\bar{S}$ is the direction of search in the design space, and the scalar α is referred to as a move parameter which,together with $\bar{S}$, determines how much the vector $\bar{X}$ is changed during the q-th iteration. An initial design defined by $\bar{X}$ must be supplied. The optimization process then proceeds in two steps. First, the direction $\bar{S}$, which improves the design, is found, and second, the scalar α , is determined which improves the design as much as possible when moving in this direction. The process is repeated until there is no further design improvement, indicating that this is the optimum attainable

design. For further details see Ref. 7.

D. COPES AND SUBROUTINE ANALIZ

In order to simplify the use of CONMIN and to further aid in the design optimization process a Control Program For Engineering Synthesis, COPES, was developed by Vanderplaats [Ref. 7]. COPES is the main program (recall that CONMIN is written in subroutine form). The user must supply an analysis subroutine with the name ANALIZ, which will calculate the various parameters. This subroutine has three segments; INPUT, EXECUTION, OUTPUT.

All parameters which may be design variables, objective functions or constraints are contained in a single labeled common block called GLOBCM.

Copes Terminology

The COPES program currently provides six specific capabilities;

1. Simple analysis, just as if COPES was not used.
2. Optimization-Minimization or maximization of one calculated function with limits imposed on other functions.
3. Sensitivity analysis- The effect of changing one or more design variables on one or more calculated functions.
4. Two-variable function space-Analysis for all specified combinations of two design variables.
5. Optimum sensitivity- The same as sensitivity analysis except that, at each step, the design is optimized with respect to the independent design variables.
6. Approximate optimization- Optimization using approximation techniques. Usually more efficient than standard optimization for up to 10 design variables or if multiple optimizations are to be performed [Ref. 7].

IV. DESCRIPTION OF THE COMPUTER PROGRAM

A. GENERAL PROGRAM ORGANIZATION

A functional block diagram of the program is presented in Fig. 4. A general description of the subroutines contained in the program is given here. Appendices A through D discuss the preparation of input data, list the important computer program nomenclature, and list the program.

B. SUBROUTINES

1. Subroutine ANALIZ

Subroutine ANALIZ organizes the basic analysis used in the optimization. It controls the reading of the initial design description and calculation of the values of the objective function, constraints, and all other parameters necessary to solve the problem. COPES/CONMIN updates the design to minimize/maximize the objective function, iterating until no further improvement in the objective function is possible without violating one of the constraints. COPES/CONMIN calls subroutine ANALIZ to obtain the function value during the optimization.

2. Subroutine INPUT

This subroutine reads all input data associated with the brake analysis. Instructions for problem deck preparation are given in appendix B.

3. Subroutine TEMPR

This subroutine calculates the heat transfer constants such as the thermal capacity of each node and the resistance of each node, determines the time increment in order to insure a stable solution, and calculates the rate of heat generation. In order to calculate the temperature of each node, it calls two subroutines. From subroutine BRAK it

obtains the deceleration needed to calculate the rate of heat generated and from subroutine TEMA it obtains the temperature rise of each node. Then it calculates the temperatures during the time that the brake is not in use. This subroutine is also capable of calculating the temperature rise of a drum when a constant rate of heat dissipation is given.

4. Subroutine TEMA

This subroutine calculates the temperature of each node. As mentioned before, the heat is generated on the inner surface, and on the outer side of the drum the heat is dissipated by convection. The formulas used were developed by the finite difference method, and are given in section II-E-2.

5. Subroutine BRAK

This subroutine calculates the torque, actuating force, and the friction moment of one shoe. It also calculates the drum volume and the deceleration of the machine. The subroutine takes into consideration a constant friction coefficient until a temperature of $90^{0}C$ is reached and a linear decrease in the friction coefficient for higher temperatures. More details are given in section II-D-4.

6. Subroutine OUTPUT

This subroutine echos the input data and prints out the thermal and mechanical information for the brake. An example of the output obtained from this subroutine is shown in Table 1 and Table 2.

V. TEST PROBLEM AND RESULTS

The computer program was tested with the data specified in Table 1. The objective function which was minimized was the volume of the drum material. Design variables were the drum width, the angle between the hinged pin and the end of the lining, the ratio of the pivot to center of rotation distance to drum radius, and the drum thickness. The side constraints (limits) on the design variables were;

	Design Variable	Lower Bound	Upper Bound
1.	(3) Width,b	0.0	80 mm.
2.	(5) Theta 2,	1.2 rad.	2.5 rad.
3.	(12) Ratio,Rd	0.1	0.9
4.	(18) Thickness,tk	40 mm.	No bound

The number in parentheses is the location of the variable in the COMMON block in the computer program.

Constraints were imposed on the actuating force F, the maximum temperature, T_{max}, on the inner surface of the drum and stopping time, t.

	Constrained Variable	Lower Bound	Upper Bound
1.	(9) Force,F	200.0 N-m	2500.0 N-m
2.	(25) Time, t	No bound	7.00 Sec.
3.	(4) Temperature,T	No bound	230.0 ^{0}C

The vehicle which weights 25700.0 Newtons is stopped four consecutive times from a velocity of 90.0 Km/hr to zero, with an acceleration period of 20.0 sec. between stops. The values of the design variables and the constraints before and after optimization are;

	Before Optimization	After Optimization
Objective Function		
Drum Volume	0.754 E-03 m^3	0.159 E-02 m^3
Design variables		
Width	0.08 m	0.08 m
Theta 2	2.10 Rad.	1.92 Rad.
a/r	0.75	0.755
Thickness	0.010 m	0.020 m
Constraints		
Actuating Force	2815.1 N	2086.3 N
Stopping time (last stop)	7.04 sec.	7.00 sec.
Temperature after last stop	348.7 °C	229.2 °C

Note that the objective function increased as a result of optimization. This is because the initial design violated constraints on stopping time and maximum temperature.

Further results are listed in Tables 1 and 2. In addition to optimization, a sensitivity analysis of the design variables and a two-variable function space analysis for width and thickness were performed. The graphical results are given in Figs. 5 through 17. The results can be summerized as follows;

a. The effect of changing the inside drum radius with all other design variables held constant;
   As shown in Figs. 5-7, for small inside drum radii the drum temperature is very high. The stopping time is long and the torque is low. Inside drum radii over 130 mm give reasonable drum temperature and stopping time, for the example considered.

b. As seen in Figs. 8-10, the effect of changing the drum width with all other design variables held constant is the same as described above.

c. The effect of changing the drum thickness with all other parameters held constant is; For a drum thickness up to 6 mm, the stopping time and drum temperature are considerably high. Over 16 mm thickness, the stopping time remains almost constant. For a small thickness the torque is very low due to the high temperatures. For thicknesses over 20 mm, the torque remains about constant.

d. The effect of changing the angle between the hinged pin and the end of the lining is; For a small θ_2 angle the stopping time is very long because the torque is low. The stopping time becomes reasonable when $\theta_2 > 1.8$ Rad. Obviously there is an increase in the drum temperature as θ_2 increases but the overall change in temperature is small.

e. From the two variable function space, Fig. 17, it can be seen that the constant volume line and the constant temperature line are almost parallel, this leads to the conclusion that for the cycle taken, the drum is a heat sink, and the amount of heat dissipated by convection during this cycle is small.

VI. TEMPERATURE RAISE - SIMPLIFIED CALCULATION

A simplified way of finding the temperature rise of the drum is by using the equation;

$$Q = \frac{W}{g} c \Delta T \tag{40}$$

and setting Q equal to the amount of heat generated using Equation (21) from section II-D-4. This equation is in common use in engineering texts (See, for example, Refs. 2 and 8). The temperature rise calculated this way, is the average temperature of the drum, and not the temperature on the interface, which can be much higher (depending on the rate of heat generated). Extreme temperature gradients cause distortion and excessive surface wear. Therefore it isn't always acceptable to use the simplified formula. From experience, it has been found that the surface wear increases dramatically as interface temperatures approach 400 to 500 ^{0}F (205 to 260 ^{0}C), [Ref. 8].

A comparison of the temperatures calculated on the inner drum surface, outer drum surface, and the average drum temperature calculated, using equation (40), is given in Fig. 18. The graph shows the temperature rise for a vehicle stopped from a velocity of 90 km/hr. From this graph, it can be seen that the drum temperature, based on equation (40), after the vehicle stopped is about the average temperature of the inner and outer surface temperatures.

The results show that the drum will reach an uniform temperature of about 58 ^{0}C, in 15 sec, after the vehicle has stopped.

Calculating the temperature with the simplified formula, can lead to errors in the time needed to stop the vehicle. Because the temperature calculated with the simplified

formula is lower than the temperature at the friction interface, the calculated friction coefficient is higher than the actual friction coefficient. Therefore the calculated stopping time will be shorter than the real stopping time. All this is true, provided the friction material behaves as assumed in section I-D-4.

Because high temperature is detrimental to both the stopping ability and the wear characteristics of the brake, it is important that the interface temperature be calculated with reasonable accuracy in design. Fig. 18 clearly shows the temperature differences resulting from the two approaches.

This difference in results is compounded when the simplified equation is used for design. Table 3 presents the design results based on the simplified approach. This design represents an apparent material savings of 27%. However, when this optimum is analysed using the finite difference heat transfer solution, the maximum temperature is 268.8 ^{0}C and the last stopping time is 7.54 sec. This time violates the constraint by 7.7%. Perhaps more importantly, the temperature at the interface of about 269 ^{0}C would surely lead to premature failure. Therefore this design is clearly too unconservative to be acceptable.

VII. CONCLUSION

In summary, a numerical optimization program is an effective way of finding a solution to an engineering problem, provided reasonable care is used in formulating the problem.

VIII. RECOMMENDATIONS FOR FUTURE INVESTIGATIONS

The study has shown the feasibility of using numerical optimization in the design of Internal-Expanding Rim Brakes with two leading shoes. Further studies on the same design may be pursued by eliminating some of the restrictions. For example;

1. To add heat dissipation by radiation.
2. To investigate drum temperatures for a drum with fins.
3. To take into consideration changes in the surface pressure as a function of friction coefficient.
4. To repeat all the calculations for a drum in which there is one trailing shoe and one leading shoe.
5. To add the effect of centrifugal forces for clutches.

```
                         TABLE NO. 1                           ANA03150
                         ----------                            ANA03160
RESULTS BEFORE OPTIMIZATION                                    ANA03170
----------------------------                                   ANA03180
                                                               ANA03190
                                                               ANA03200
     THETA1     = 0.15000E+00   RAD.                           ANA03210
     THETA2     = 0.21000E+01   RAD.                           ANA03220
     THETA A    = 0.15708E+01   RAD.                           ANA03230
     PRESSUREA= 0.68950E+06     N/M^2                          ANA03240
     WIDTH      = 0.80000E-01   M                              ANA03250
     INSIDE RADIUS = 0.14500E+00  M                            ANA03260
     DRUM THICKNESS= 0.10000E-01  M                            ANA03270
                                                               ANA03280
                                                               ANA03290
     CONDUCTIVITY COEFF.= 0.50000E+02   W/M-^0C                ANA03300
     CONVECTION COEFF.  = 0.30000E+02   W/M^2-^0C              ANA03310
     SPEC. HEAT COEFF.  = 0.47000E+03   J/KG-^0C               ANA03320
     MAX.TEMP.DIFFERENCE= 0.70000E+03  ^0C                     ANA03330
     COLD FRICTION COEFF= 0.35000E+00                          ANA03340
     HOT FRICTION COEFF.= 0.15000E+00                          ANA03350
     INITIAL TEMPERATURE= 0.30000E+02  ^0C                     ANA03360
     DRUM DENSITY       = 0.78000E+04   KG /M^3                ANA03370
     CAR WEIGHT         = 0.26700E+05   N                      ANA03380
```

```
WHEEL RADIUS        = 0.40000E+00    M                                     ANA03390
TIME STEP           = 0.10000E-01    SEC.                                  ANA03400
                                                                           ANA03410
                                                                           ANA03420
FRICTION MOMENT    = 0.57074E+03    N-M                                    ANA03430
NORMAL MOMENT      = 0.11018E+04    N-M                                    ANA03440
ACTUATING FORCE    = 0.28151E+04    N                                      ANA03450
DIST. FORCE-PIVOT  = 0.18866E+00    M                                      ANA03460
DIST. CENTER-PIVOT= 0.10875E+00     M                                      ANA03470
RATIO A/R          = 0.75000E+00                                           ANA03480
MIU       = 0.27888E+00                                                    ANA03490
TORQUE    = 0.48308E+03   N-M                                              ANA03500
                                                                           ANA03510
TOT.TIME     INSIDE TEMP.     OUTSIDE TEMP.                                ANA03520
   SEC.           0C               0C                                      ANA03530
  85.44        0.3388E+03      0.3305E+03                                  ANA03540
                                                                           ANA03550
MAXIMUM INSIDE DRUM TEMP.= 0.34869E+03   0C                                ANA03560
                                                                           ANA03570
STOPPING TIME=      5.750    SEC.                                          ANA03580
STOPPING TIME=      6.130    SEC.                                          ANA03590
STOPPING TIME=      6.560    SEC.                                          ANA03600
STOPPING TIME=      7.040    SEC.                                          ANA03610
```

```
                             TABLE NO. 2
                             -----------

FINAL OPTIMIZATION INFORMATION

-------------------------------
 THERE ARE    2 ACTIVE CONSTRAINTS
 CONSTRAINT NUMBERS ARE
     4    6
 THERE ARE    0 VIOLATED CONSTRAINTS
 THERE ARE    1 ACTIVE SIDE CONSTRAINTS
 TERMINATION CRITERION
      ABS(1-OBJ(I-1)/OBJ(I)) LESS THAN DELFUN FOR  2 ITERATIONS
      ABS(OBJ(I)-OBJ(I-1))   LESS THAN DABFUN FOR  2 ITERATIONS


 OBJECTIVE FUNCTION
 GLOBAL LOCATION   27     FUNCTION VALUE 0.15924E-02
```

```
DESIGN VARIABLES                                                            ANA03850
                                                                            ANA03860
         D. V.     GLOBAL      LOWER                           UPPER        ANA03870
  ID      NO.     VAR. NO.     BOUND           VALUE           BOUND        ANA03880
   1       1           3    0.0            0.80000E-01    0.80000E-01  ANA03890
   2       2           6    0.12000E+01    0.19251E+01    0.25000E+01  ANA03900
   3       3          12    0.10000E+00    0.75513E+00    0.90000E+00  ANA03910
   4       4          18    0.40000E-02    0.20411E-01    0.11000E+16  ANA03920
                                                                            ANA03930
 DESIGN CONSTRAINTS                                                         ANA03940
                                                                            ANA03950
           GLOBAL      LOWER                          UPPER                 ANA03960
    ID    VAR. NO.     BOUND          VALUE           BOUND                 ANA03970
     1         9    0.20000E+03   0.20863E+04   0.25000E+04                 ANA03980
     3         4    0.30000E+02   0.22922E+03   0.23000E+03                 ANA03990
     5        26    0.0           C.70000E+01   0.70000E+01                 ANA04000
                                                                            ANA04010
 STOPPING TIME=     6.380    SEC.                                           ANA04020
 STOPPING TIME=     6.570    SEC.                                           ANA04030
 STOPPING TIME=     6.780    SEC.                                           ANA04040
 STOPPING TIME=     7.000    SEC.                                           ANA04050
```

```
                                                                ANA04080
THETA1     = 0.15000E+00    RAD.                                ANA04090
THETA2     = 0.19251E+01    RAD.                                ANA04100
THETA A    = 0.15708E+01    RAD.                                ANA04110
PRESSUREA= 0.68950E+06     N/M^2                                ANA04120
WIDTH      = 0.79070E-01    M                                   ANA04130
INSIDE RADIUS = 0.14500E+00    M                                ANA04140
DRUM THICKNESS= 0.19872E-01    M                                ANA04150
                                                                ANA04160
                                                                ANA04170
CONDUCTIVITY COEFF.= 0.50000E+02   W/M-^0C                      ANA04180
CONVECTION COEFF.  = 0.30000E+02   W/M^2-^0C                    ANA04190
SPEC. HEAT COEFF.  = 0.47000E+03   J/KG.-^0C                    ANA04200
MAX.TEMP.DIFFERENCE= 0.70000E+03  ^0C                           ANA04210
COLD FRICTION COEFF= 0.35000E+00                                ANA04220
HOT FRICTION COEFF.= 0.15000E+00                                ANA04230
INITIAL TEMPERATURE= 0.30000E+02  ^0C                           ANA04240
DRUM DENSITY       = 0.78000E+04   KG /M^3                      ANA04250
CAR WEIGHT         = 0.26700E+05   N                            ANA04260
WHEEL RADIUS       = 0.40000E+00   M                            ANA04270
TIME STEP          = 0.10000E+00   SEC.                         ANA04280
```

```
                                                                ANA04310
                                                                ANA04320
FRICTION MOMENT     = 0.60984E+03     N-M                       ANA04330
NORMAL MOMENT       = 0.98441E+03     N-M                       ANA04340
ACTUATING FORCE     = 0.20863E+04     N                         ANA04350
DIST. FORCE-PIVOT   = 0.17971E+00     M                         ANA04360
DIST. CENTER-PIVOT= 0.10949E+00       M                         ANA04370
RATIO A/R           = 0.75513E+00                               ANA04380
MIU       = 0.31671E+00                                         ANA04390
TORQUE    = 0.49059E+03   N-M                                   ANA04400
                                                                ANA04410
TOT.TIME    INSIDE TEMP.   OUTSIDE TEMP.                        ANA04420
  SEC.          0C             0C                               ANA04430
  86.69      0.2064E+03     0.1627E+03                          ANA04440
                                                                ANA04450
MAXIMUM INSIDE DRUM TEMP.= 0.22922E+03  0C                      ANA04460
```

```
                        TABLE NO. 3
                        -----------
              OPTIMIZED RESULTS-SIMPLIFIED WAY
              --------------------------------

FINAL OPTIMIZATION INFORMATION

 THERE ARE    1 ACTIVE CONSTRAINTS
 CONSTRAINT NUMBERS ARE
     4
 THERE ARE    0 VIOLATED CONSTRAINTS
 THERE ARE    0 ACTIVE SIDE CONSTRAINTS
 TERMINATION CRITERION
      ABS(OBJ(I)-OBJ(I-1))   LESS THAN DABFUN FOR  2 ITERATIONS

 NUMBER OF ITERATIONS =    4

 OBJECTIVE FUNCTION
 GLOBAL LOCATION    27     FUNCTION VALUE 0.11610E-02
```

DESIGN VARIABLES

ID	D. V. NO.	GLOBAL VAR. NO.	LOWER BOUND	VALUE	UPPER BOUND
1	1	3	0.0	0.77081E-01	0.80000E-01
2	2	6	0.12000E+01	0.21187E+01	0.25000E+01
3	3	12	0.10000E+00	0.74590E+00	0.90000E+00
4	4	18	0.40000E-02	0.15685E-01	0.11000E+16

DESIGN CONSTRAINTS

ID	GLOBAL VAR. NO.	LOWER BOUND	VALUE	UPPER BOUND
1	9	0.20000E+03	0.24170E+04	0.25000E+04
3	8	0.30000E+02	0.23000E+03	0.23000E+03
5	26	0.0	0.65000E+01	0.70000E+01

STOPPING TIME= 0.5600E+01 THE TEMPERATURE IS= 0.80005E+02
STOPPING TIME= 0.5900E+01 THE TEMPERATURE IS= 0.13001E+03
STOPPING TIME= 0.6200E+01 THE TEMPERATURE IS= 0.18000E+03
STOPPING TIME= 0.6500E+01 THE TEMPERATURE IS= 0.23000E+03

THETA1 =	0.15000E+00	RAD.	ANA04940
THETA2 =	0.21187E+01	RAD.	ANA04950
THETA A =	0.15708E+01	RAD.	ANA04960
PRESSUREA=	0.68950E+06	N/M^2	ANA04970
WIDTH =	0.77081E-01	M	ANA04980
INSIDE RADIUS =	0.14500E+00	M	ANA04990
DRUM THICKNESS=	0.15685E-01	M	ANA05000
			ANA05010
CONDUCTIVITY COEFF.=	0.50000E+02	$W/M-{}^0C$	ANA05020
CONVECTION COEFF. =	0.30000E+02	$W/M^2-{}^0C$	ANA05030
SPEC. HEAT COEFF. =	0.47000E+03	$J/KG.-{}^0C$	ANA05040
MAX.TEMP.DIFFERENCE=	0.70000E+03	^0C	ANA05050
COLD FRICTION COEFF=	0.35000E+00		ANA05060
HOT FRICTION COEFF.=	0.15000E+00		ANA05070
INITIAL TEMPERATURE=	0.30000E+02	^0C	ANA05080
DRUM DENSITY =	0.78000E+04	$KG./M^3$	ANA05090
CAR WEIGHT =	0.26700E+05	N	ANA05100
WHEEL RADIUS =	0.40000E+00	M	ANA05110
TIME STEP =	0.10000E+00	SEC.	ANA05120

```
   FRICTION MOMENT     = 0.61469E+03     N-M                                   ANA05150
   NORMAL MOMENT       = 0.10731E+04     N-M                                   ANA05160
   ACTUATING FORCE     = 0.24170E+04     N                                     ANA05170
   DIST. FORCE-PIVOT   = 0.18964E+00     M                                     ANA05180
   DIST. CENTER-PIVOT= 0.10873E+00       M                                     ANA05190
   RATIO A/R           = 0.74990E+00                                           ANA05200
   MIU       = 0.31000E+00                                                     ANA05210
   TURQUE    = 0.52296E+03    N-M                                              ANA05220
                                                                               ANA05230
 THE FINAL DRUM TEMPERATURE IS= 0.23000E+03   °C                               ANA05240
```

```
                          TABLE NO. 4
                          ----------
          RESULTS WITH DIMENSIONS ACHIEVED WITH THE SIMPLIFIED WAY
          --------------------------------------------------------

      THETA 1   = 0.15000E+00   RAD.
      THETA 2   = 0.19251E+01   RAD.
      THETA A   = 0.15708E+01   RAD.
      PRESSUREA= 0.68950E+06   N/M2
      WIDTH     = 0.77081E-01   M
      INSIDE RADIUS = 0.14500E+00   M
      DRUM THICKNESS= 0.15690E-01   M


      CONDUCTIVITY COEFF.= 0.50000E+02   W/M-°C
      CONVECTION COEFF.  = 0.30000E+02   W/M2-°C
      SPEC. HEAT COEFF.  = 0.47000E+03   J/KG-°C
      MAX.TEMP.DIFFERENCE= 0.70000E+03  °C
      COLD FRICTION COEFF= 0.35000E+00
      HOT FRICTION COEFF.= 0.15000E+00
      INITIAL TEMPERATURE= 0.30000E+02  °C
      DRUM DENSITY       = 0.78000E+04   KG/M3
      CAR WEIGHT         = 0.26700E+05   N
      WHEEL RADIUS       = 0.40000E+00   M
```

```
TIME STEP            = 0.10000E-01    SEC.                                   ANA05510
                                                                             ANA05520
                                                                             ANA05530
FRICTION MOMENT     = 0.56543E+03     N-M                                    ANA05540
NORMAL MOMENT       = 0.94853E+03     N-M                                    ANA05550
ACTUATING FORCE     = 0.21317E+04     N                                      ANA05560
DIST. FORCE-PIVOT   = 0.17971E+00     M                                      ANA05570
DIST. CENTER-PIVOT= 0.10949E+00       M                                      ANA05580
RATIO A/R           = 0.75513E+00                                            ANA05590
MIU       = 0.30494E+00                                                      ANA05600
TORQUE    = 0.45514E+03   N-M                                                ANA05610
                                                                             ANA05620
TOT.TIME     INSIDE TEMP.    OUTSIDE TEMP.                                   ANA05630
   SEC.           °C              °C                                         ANA05640
  88.25       0.2476E+03     0.2225E+03                                      ANA05650
                                                                             ANA05660
MAXIMUM INSIDE DRUM TEMP.= 0.26838E+03   °C                                  ANA05670
STOPPING TIME=      6.630   SEC.                                             ANA05680
STOPPING TIME=      6.910   SEC.                                             ANA05690
STOPPING TIME=      7.210   SEC.                                             ANA05700
STOPPING TIME=      7.540   SEC.                                             ANA05710
```

APPENDIX A

LIST OF PARAMETERS

A complete listing and description of all variables used in the program, is not practical. The variables listed in this appendix are common to several subroutines of the program and will assist the reader in a study of the program. The Global location is the location of the parameter in the common block called GLOBCM. This common block is the means by which information is transfered between the subroutines and the COPES/CONMIN program.

Global Location	Fortran Name	Math. Symbol	Definition
1	RI	r	Inside drum radius (m)
2	T	T_0	Torque of one shoe (N-m)
3	WDTH	b	Drum width (m)
4	PRSA	p	Pressure between lining and drum (N/m^2)
5	TETA1	θ_1	The angle between the hinged pin and the (Rad.) begining of the lining
6	TETA2	θ_2	The angle between the hinged pin and the end of the lining (Rad.)
7	FRMNT	Mf	Friction moment (N-m)
8	ANMRT	Mn	Normal momemt (N-m)
9	ACFRC	F	Actuating force (N)
10	C	d	Distance from actuating force to the hinged pin(m)

11	Q	Q	Heat generated (J/sec.)
12	RD	a	Distance from pivot to center of rotation (m)
13	CMIU	μ_c	Cold friction coefficient
14	HMIU	μ_h	Hot friction coefficient
15	AMIU	μ	Friction coefficient at any temperature
16	SRFC		Drum surface area (m^2)
17	RO		Outside drum radius (m)
18	THK	tk	Drum thickness (m)
19	DX		An incremental thickness (m)
20	RTIER	R	Wheel radius (m)
21	W	W	Car's weight (N)
22	DCCE	dc	Deceleration ($m/sec.^2$)
23	TOT		Total time (sec.)
24	ECEN	a	Ecentricity (m)
25	NWRT		Write statement control
26	TIME	t	Time (sec.)
27	VOL	V_0	Drum volume (m^3)
28-32	TEPL(5)	T	Temperature at time p+1 (sec.)
33	NWR		Write statement control
34	NWRA		Write statement control
35	NWRQ		Write statement control
36	NEL		Number of elements
37	NSEG		Number of segments
38	PI	π	Constant
39	G	g	Gravitational constant
40	K	k	Thermal conductivity ($J/m-{}^0C$)
41	HCNV	h	Convection heat coeff. ($W/m^2-{}^0C$).
42	SPHT	c	Specific heat ($J/Kg-{}^0C$)

43	RHO	ρ	Density (Kg./m^3)
44	DTAU		Time increment (sec.)
45	DFTM	T	Max. temp. difference(^{0}C)
46-50	TEMP	T	Temp. at time p (sec.)
51-57	RES		Heat resistance (^{0}C/J)
58-63	TC	C	Heat capacity (J/^{0}C)
64	BIO	Bi	Biot moduli
65	FUR	Fo	Fourier moduli
66-72	NVT		Control parameter
73-79	VT		Control parameter
80	NELO		Number of elements+1
81	NELT		Number of elements+2
82	TETAA	θ_a	The angle at which the pressure between the lining and drum is maximum. (Rad.)
83	ACOF		Constant
84	TINI	T	Initial temperature(^{0}C)
85	ZMAN	t	Time increment (sec.)
86	NSHU		Number of shoes

APPENDIX B

INSTRUCTIONS FOR PROBLEM DATA PREPARATION

Although the procedure is straight forward, preparation of input data for the program requires attention. Errors are easy to make and difficult to locate. Input data is described here for the brake analysis. For instructions on data preparation for optimization see Ref. 7. Input data should, in general, follow the steps outlined below. The use of the standard FORTRAN Eighty Columm Coding Sheet is recommended. Integer constants must be right justified in the appropriate field. There are eight input cards, read by subroutine INPUT, to describe the initial design, material properties and constants. Card format is given in parenthesis followed by specific instructions where necessary.

1. First Card (I10) - Duty cycle information.
   - Cols 1-10 : Total number of consecutives stops and accelerations (NSEG)
2. Second Card (I10,3F10.0) - Duty cycle information.
   - a. Cols 1-10 : Control number.
     1 means-deceleration,
     2 means-brake not in use,
   - b. Cols 11-20 : Velocity at start of deceleration.
   - c. Cols 21-30 : The velocity at the end of the deceleration.
   - d. Cols 31-40 : The time the brake is not in use.
3. Third Card (5I10) - Thermal analysis information.
   - a. Cols 1-10 : Number of nodes (NEL).
   - b. Cols 11-20 : An integer number that controls the amount of printout when detailed output is required during the vehicle deceleration. The amount of

lines written, depends on the stopping time and time increment. (NWR).

c. Cols 21-30 : An integer number that controls the amount of printout when detailed output of the temperatures is required during the period that the vehicle is not in use. The amount of lines written depends on the period length that the brakes are not used (NWRA).

d. Cols 31-40 : An integer number that controls the amount of printout when detailed output of the temperatures are required during the period of constant heat generation. (NWRQ).

e. Cols 41-50 : Number of braking shoes in the machine.

4. Fourth Card (7F10.0) - Brake dimensions.

a. Cols 1-10 : Inside drum radius (RI).

b. Cols 11-20 : Drum width (WDTH).

c. Cols 21-30 : Drum thickness (THK).

d. Cols 31-40 : Ratio of distance from pivot to center of rotation and inside radius (RD).

e. Cols 41-50 : Drum density (RHO).

f. Cols 51-60 : Angle between hinged pin and the begining of the lining (TETA1).

g. Cols 61-70 : Angle between hinged pin and the end of the lining (TETA2).

5. Fifth Card (7F10.0) - Thermal and friction information.

a. Cols 1-10 : Heat conduction coefficient (K). (real number).

b. Cols 11-20 : Heat convection coefficient (HCNV).

c. Cols 21-30 : Specific heat of the drum (SPHT).

d. Cols 31-40 : Max. temperature difference between cold friction coefficient and hot friction coefficient (DFTM).

e. Cols 41-50 : Cold friction coefficient (CMIV).

f. Cols 51-60 : Hot friction coefficient (HMIV).

g. Cols 61-70 : Initial temperature (TINI).

6. Sixth Card (2F10.0) - Machine information.

a. Cols 1-10 : Vehicles weight (W).

b. Cols 11-20 : Wheel radius (RTIER).

7. Seventh Card (5F10.0) - Analysis constants.

a. Cols 1-10 : Maximum pressure between lining and drum (PRSA).

b. Cols 11-20 : Constant 3.1415927

c. Cols 21-30 : Gravitational constant (G).

d. Cols 31-40 : Increment of time (ZMAN).

e. Cols 41-50 : The angle of maximum pressure (TETAA).

8. Eight Card (I10.0) - Print control.

Cols 1-10 : An integer number can be zero or 1. If zero (or a blank card) - only the final results are printed. If 1- the temperature at time increments are printed.

```
                      APPENDIX C                                  ANA05740
                                                                  ANA05750
                 STANDARD DECK STRUCTURE                          ANA05760
COPES DATA                                                        ANA05770
                                                                  ANA05780
CLUTCH OPTIMIZATION                                               ANA05790
$  DATA BLOCK B                                                   ANA05800
$      NCALC    NDV       NSV       N2VAR                         ANA05810
4,4,4,4                                                           ANA05820
$  DATA BLOCK C                                                   ANA05830
$     IPRINT    ITMAX     ICNDIR     NSCAL      ITRM              ANA05840
2,20,0,5,2                                                        ANA05850
$  DATA BLOCK D                                                   ANA05860
0.0                                                               ANA05870
0.0                                                               ANA05880
$  DATA BLOCK E                                                   ANA05890
$      NDVTOT    IOBJ     SGNOPT                                  ANA05900
0,27,-1.0                                                         ANA05910
$  DATA BLOCK F                                                   ANA05920
$     VLB      VUB                                                ANA05930
0.0,0.08                                                          ANA05940
1.2,2.5                                                           ANA05950
0.1,0.9                                                           ANA05960
0.004,1.0+20                                                      ANA05970
```

```
$  DATA BLOCK G                                              ANA05980
$       NDSGN       IDSGN       AMULT                        ANA05990
1,3,1.0                                                      ANA06000
2,6,1.0                                                      ANA06010
3,12,1.0                                                     ANA06020
4,18,1.0                                                     ANA06030
$  DATA BLOCK H                                              ANA06040
$      NCONS                                                 ANA06050
3                                                            ANA06060
$  DATA BLOCK I                                              ANA06070
$       ICON       JCON       LCON                           ANA06080
9                                                            ANA06090
200.0,0.0,2500.0,0.0                                         ANA06100
4                                                            ANA06110
30.0,0.0,230.0,0.0                                           ANA06120
26                                                           ANA06130
0.0,0.0,7.0,0.0                                              ANA06140
$  DATA BLOCK P                                              ANA06150
4                                                            ANA06160
2,26,28,4                                                    ANA06170
$  DATA BLOCK Q                                              ANA06180
$ INSIDE RADIUS                                              ANA06190
1,12                                                         ANA06200
0.145,0.07,0.08,0.09,0.10,0.11,0.12,0.13                     ANA06210
```

```
0.15,0.16,0.17,0.145                                          ANA06220
$  WIDTH                                                      ANA06230
3,13                                                          ANA06240
0.08,0.02,0.03,0.04,0.05,0.06,0.07,0.08                       ANA06250
0.09,0.10,0.12,0.13,0.14                                      ANA06260
$  TETA2                                                      ANA06270
6,13                                                          ANA06280
1.9251,C.4,0.6,0.8,1.0,1.25,1.5,1.75                          ANA06290
2.0,2.25,2.5,2.75,3.0                                         ANA06300
$  THIKNESS                                                   ANA06310
18,15                                                         ANA06320
0.020411,0.003,0.004,0.005,0.006,0.007,0.008,0.01             ANA06330
0.012,0.014,0.016,0.018,0.020,0.022,0.024                     ANA06340
$  DATA BLOCK R                                               ANA06350
6,12,3,9                                                      ANA06360
$  DATA BLOCK S                                               ANA06370
9,26,27,28                                                    ANA06380
$  DATA BLOCK T                                               ANA06390
0.4,0.6,0.8,1.0,1.25,1.5,1.75,2.0                             ANA06400
2.25,2.5,2.75,3.0                                             ANA06410
$  DATA BLOCK U                                               ANA06420
0.06,0.07,0.08,0.09,0.1,0.12,0.13,0.14,0.15                   ANA06430
$  DATA BLOCK V                                               ANA06440
END                                                           ANA06450
```

ANALIZ DATA

```
                                                                      ANA06480
                                                                      ANA06490
                                                                      ANA06500
                                                                      ANA06510
      7                                                               ANA06520
      1      25.                                                      ANA06530
      2                      20.                                      ANA06540
      1      25.                                                      ANA06550
      2                      20.                                      ANA06560
      1      25.                                                      ANA06570
      2                      20.                                      ANA06580
      1      25.                                                      ANA06590
      5      65      500     200      8                               ANA06600
  0.145  0.080000 0.02041 0.75513  7800.0      0.15     1.925         ANA06610
   50.0     30.0   470.0   700.0    0.35      0.15        30.0       ANA06620
26700.0      0.4                                                      ANA06630
689500.03.14159265   9.8     0.1     1.0                              ANA06640
                                                                      ANA06650
```

APPENDIX D

PROGRAM LISTING

```
C
C           THIS SUBROUTINE ORGANIZES THE BRAKE ANALYSIS
C
C
      SUBROUTINE ANALIZ (ICALC)
      DIMENSION TEMP(50),TEPL(50),A(50),RES(50),TC(50),NVT(20),VT(20,4)
      COMMON /GLOBCM/ RI,T,WDTH,TMAX,TETA1,TETA2,FRMNT,ANRMNT,ACFRC,C,Q,
     1RD,CMIU,HMIU,AMIU,SRFC,RO,THK,DX,RTIER,W,DCCE,TOT,ECEN,NWRT,TIME,V
     2OL,TEPL,NWR,NWRA,NWRQ,NEL,NSEG,PI,G,K,HCNV,SPHT,RHO,DTAU,DFTM,TEMP
     3,RES,TC,BIO,FUR,NVT,VT,NELO,NELT,TETAA,ACOF,TINI,ZMAN,NSHU,PRSA
C
C           BY M. PEER                   FEB. 1981
C           NPGS               MONTEREY CA. 93940
C           ISRAELI ARMY  MILITARY P.O.B. 2128
C           ISRAEL
      TOT=0.0
      TMAX=0.0
      IF (ICALC.EQ.1) CALL INPUT
      SAVE=ZMAN
      DTAU=SAVE
      CALL TEMPR (ICALC)
      IF (ICALC.EQ.1.OR.ICALC.EQ.3) CALL OUTPUT (ICALC)
      RETURN
      END
```

```
C                             SUBROUTINE INPUT
C
C
      SUBROUTINE INPUT
      DIMENSION TEMP(50),TEPL(50),A(50),RES(50),TC(50),NVT(20),VT(20,4)
      COMMON /GLOBCM/ RI,T,WDTH,TMAX,TETA1,TETA2,FRMNT,ANRMNT,ACFRC,C,Q,
     1RD,CMIU,HMIU,AMIU,SRFC,RO,THK,DX,RTIER,W,DCCE,TOT,ECEN,NWRT,TIME,VA
     2OL,TEPL,NWR,NWRA,NWRQ,NEL,NSEG,PI,G,K,HCNV,SPHT,RHO,DTAU,DFTM,TEMP
     3,RES,TC,BIO,FUR,NVT,VT,NELO,NELT,TETAA,ACOF,TINI,ZMAN,NSHU,PRSA
      REAL K
      READ (5,60) NSEG
      DO 10 I=1,NSEG
      READ (5,60) NVT(I),(VT(I,J),J=1,4)
   10 CONTINUE
      READ (5,20) NEL,NWR,NWRA,NWRQ,NSHU
      READ (5,30) RI,WDTH,THK,RD,RHO,TETA1,TETA2
      READ (5,30) K,HCNV,SPHT,DFTM,CMIU,HMIU,TINI
      READ (5,30) W,RTIER
      READ (5,30) PRSA,PI,G,ZMAN,TETAA
      READ (5,20) NWRT
      RETURN
C
   20 FORMAT (5I10)
   30 FORMAT (7F10.0)
   60 FORMAT (I10,4F10.0,I10)
      END
```

```
C                       SUBROUTINE TEMPR                                        ANA00630
C                                                                               ANA00640
C                                                                               ANA00650
C   THIS SUBROUTINE CALCULATES THE TEMPERATURES                                 ANA00660
C                                                                               ANA00670
C                                                                               ANA00680
      SUBROUTINE TEMPR (ICALC)                                                  ANA00690
      DIMENSION TEMP(50),TEPL(50),A(50),RES(50),TC(50),NVT(20),VT(20,4)         ANA00700
      COMMON /GLOBCM/ RI,T,WDTH,TMAX,TETA1,TETA2,FRMNT,ANRMNT,ACFRC,C,Q,ANA00710
     1RD,CMIU,HMIU,AMIU,SRFC,RO,THK,DX,RTIER,W,DCCE,TOT,ECEN,NWRT,TIME,VANA00720
     2OL,TEPL,NWR,NWRA,NWRQ,NEL,NSEG,PI,G,K,HCNV,SPHT,RHO,DTAU,DFTM,TEMPANA00730
     3,RES,TC,BIO,FUR,NVT,VT,NELO,NELT,TETAA,ACOF,TINI,ZMAN,NSHU,PRSA           ANA00740
      REAL K                                                                    ANA00750
      DX=THK/NEL                                                                ANA00760
      NELO=NEL+1                                                                ANA00770
      NELT=NEL+2                                                                ANA00780
      DO 10 I=1,NELT                                                            ANA00790
      TEMP(I)=TINI                                                              ANA00800
10    CONTINUE                                                                  ANA00810
      ARW=6.283185307*WDTH                                                      ANA00820
      A(1)=ARW*(RI+0.25*DX)                                                     ANA00830
      DO 20 I=2,NEL                                                             ANA00840
      A(I)=ARW*(RI+(I-1)*DX)                                                    ANA00850
20    CONTINUE                                                                  ANA00860
      A(NELO)=ARW*(RI+(NELO-1.25)*DX)                                           ANA00870
C                                                                               ANA00880
C   HEAT RESISTANCE                                                             ANA00890
C                                                                               ANA00900
      DXK=DX/K                                                                  ANA00910
      DO 30 I=1,NEL                                                             ANA00920
      RES(I)=DXK/A(I)                                                           ANA00930
30    CONTINUE                                                                  ANA00940
      RES(NELO)=DXK/A(NELO)                                                     ANA00950
      RES(NELT)=1.0/(HCNV*A(NELO))                                              ANA00960
C                                                                               ANA00970
C   HEAT CAPACITY                                                               ANA00980
C                                                                               ANA00990
      TCM=RHO*SPHT*WDTH*DX*3.14159265                                           ANA01000
      TC(1)=TCM*(RI+0.25*DX)                                                    ANA01010
      DO 40 I=2,NEL                                                             ANA01020
      TC(I)=2.*TCM*(RI+(I-1)*DX)                                                ANA01030
40    CONTINUE                                                                  ANA01040
      TC(NELO)=TCM*(RI+(NELO-1.25)*DX)                                          ANA01050
C                                                                               ANA01060
C   STABILITY- TIME INTERVAL                                                    ANA01070
C                                                                               ANA01080
      STAB1=TC(1)*RES(1)                                                        ANA01090
      IF(DTAU.GT.STAB1) DTAU=STAB1                                              ANA01100
```

```
C     STAB2=TC(2)*RES(2)*RES(3)/(RES(2)+RES(3))
      STAB2=0.5*TC(2)*RES(2)
      ALPHA=K/(RHO*SPHT)
      STAB3=DX*DX/(2.*ALPHA*(1.+HCNV*DX/K))
      IF(DTAU.GT.STAB2) DTAU=STAB2
C     STAB3=TC(NELO)*RES(NELO)*RES(NELT)/(RES(NELO)+RES(NELT))
      IF(DTAU.GT.STAB3) DTAU=STAB3
C
C   BIOT MODULUS
C
      BIO=HCNV*DX/K
C
C   FOURIER MODULUS
C
      FUR=K*DTAU/(RHO*SPHT*DX*DX)
C
C
C
C   ENERGY GENERATED PER UNIT TIME
C
      DO 160 ISEG=1,NSEG
      IF (NVT(ISEG).EQ.2) GO TO 80
      IF (NVT(ISEG).EQ.3) GO TO 120
      V1=VT(ISEG,1)
      IF (NWRT.EQ.1) WRITE (6,170)
      N=0
      VCON=VT(ISEG,1)
50    CONTINUE
      CALL BRAK (ICALC)
      N=N+1
      TIME=DTAU*N
      V2=VCON-DCCE*DTAU
      Q=ACOF*V2
      CALL TEMA (ICALC)
      TOT=TOT+DTAU
      IF (N.EQ.1.AND.NWRT.EQ.1) WRITE (6,180) DCCE
      IF (MOD(N,NWR).EQ.1) GO TO 60
      GO TO 70
60    IF (NWRT.EQ.1) WRITE (6,200) TOT,TEPL(1),TEPL(NELO)
70    IF (V2.LE.VT(ISEG,2).AND.ICALC.EQ.3) WRITE (6,190) TIME
      IF (V2.LE.VT(ISEG,2)) GO TO 160
      IF (AMIU.LE.0.01) GO TO 160
      VCON=V2
      GO TO 50
C
C    BRAKE NOT IN USE
C
80    CONTINUE
```

```
      IF (NWRT.EQ.1) WRITE (6,200) TOT,TEPL(1),TEPL(NELO)
      Q=0
      N=0
      IF (NWRT.EQ.1) WRITE (6,210)
90    CONTINUE
      N=N+1
      TIME=DTAU*N
      CALL TEMA (ICALC)
      TOT=TOT+DTAU
      IF (MOD(N,NWRA).EQ.1) GO TO 100
      GO TO 110
100   IF (NWRT.EQ.1) WRITE (6,200) TOT,TEPL(1),TEPL(NELO)
110   IF (TIME.GE.VT(ISEG,3)) GO TO 160
      GO TO 90
120   Q=VT(ISEG,4)
      IF (NWRT.EQ.1) WRITE (6,200) TOT,TEPL(1),TEPL(NELO)
      IF (NWRT.EQ.1) WRITE (6,220) Q
      N=0
130   CONTINUE
      N=N+1
      TIME=DTAU*N
      CALL TEMA (ICALC)
      CALL BRAK (ICALC)
      TCT=TOT+DTAU
      IF (MOD(N,NWRQ).EQ.1) GO TO 140
      GO TO 150
140   IF (NWRT.EQ.1) WRITE (6,200) TOT,TEPL(1),TEPL(NELO)
150   IF (TIME.GE.VT(ISEG,3)) GO TO 160
      GO TO 130
160   CONTINUE
      RETURN
C
170   FORMAT (/,5X,13HDECCELERATION,/)
180   FORMAT (/,20X,20HTHE DECELARATION IS=,F7.3,/)
190   FORMAT (/,5X,14HSTOPPING TIME=,F10.3)
200   FORMAT (2X,F7.2,2X,3E13.4)
210   FORMAT (/,5X,16HBRAKE NOT IN USE,/)
220   FORMAT (/,5X,29HCONSTANT HEAT DISSIPATION  Q=,E12.5,/)
      END
```

```
C                    SUBROUTINE TEMA                                    ANA02000
C                                                                       ANA02010
C                                                                       ANA02020
C  THIS SUBROUTINE CALCULATES THE TEMPERATURE IN EVERY ELEMENT          ANA02030
C                                                                       ANA02040
C                                                                       ANA02050
      SUBROUTINE TEMA (ICALC)                                           ANA02060
      DIMENSION TEMP(50),RES(50),TC(50),A(50),TEPL(50),NVT(20),VT(20,4) ANA02070
      COMMON /GLOBCM/ RI,T,WDTH,TMAX,TETA1,TETA2,FRMNT,ANRMNT,ACFRC,C,Q,ANA02080
     1RD,CMIU,HMIU,AMIU,SRFC,RO,THK,DX,RTIER,W,DCCE,TOT,ECEN,NWRT,TIME,VANA02090
     2OL,TEPL,NWR,NWRA,NWRQ,NEL,NSEG,PI,G,K,HCNV,SPHT,RHO,DTAU,DFTM,TEMPANA02100
     3,RES,TC,BIO,FUR,NVT,VT,NELO,NELT,TETAA,ACOF,TINI,ZMAN,NSHU,PRSA   ANA02110
      TEPL(1)=2.*Q*DTAU/TC(1)+(1.0-2.0*FUR)*TEMP(1)+2.0*FUR*TEMP(2)     ANA02120
      IF(TEPL(1).GE.TMAX) TMAX=TEPL(1)                                  ANA02130
      DO 10 I=2,NEL                                                     ANA02140
      TEPL(I)=FUR*(TEMP(I-1)+TEMP(I+1)+(1.0/FUR-2.)*TEMP(I))            ANA02150
10    CONTINUE                                                          ANA02160
      TEPL(NELO)=2.*FUR*(TEMP(NEL)+BIO*TEMP(NELT)+(1./(2.*FUR)-BIO-1.)*TANA02170
     1EMP(NELO))                                                        ANA02180
      DO 20 L=1,NELO                                                    ANA02190
      TEMP(L)=TEPL(L)                                                   ANA02200
20    CONTINUE                                                          ANA02210
      RETURN                                                            ANA02220
      END                                                               ANA02230
```

```
C                    SUBROUTINE BRAK
C
C   THIS SUBROUTINE CALCULATES THE TORQUE AND  ACTUATING FORCE
C
C
      SUBROUTINE BRAK (ICALC)
      DIMENSION TEMP(50),TEPL(50),A(50),RES(50),TC(50),NVT(20),VT(20,4)
      COMMON /GLOBCM/ RI,T,WDTH,TMAX,TETA1,TETA2,FRMNT,ANRMNT,ACFRC,C,Q,
     1RD,CMIU,HMIU,AMIU,SRFC,RO,THK,DX,RTIER,W,DCCE,TOT,ECEN,NWRT,TIME,V
     2OL,TEPL,NWR,NWRA,NWRQ,NEL,NSEG,PI,G,K,HCNV,SPHT,RHO,DTAU,DFTM,TEMP
     3,RES,TC,BIO,FUR,NVT,VT,NELO,NELT,TETAA,ACOF,TINI,ZMAN,NSHU,PRSA
      IF (TETA2.GE.1.5708) TETAA=1.57079632
      IF (TETA2.LT.1.5708) TETAA=TETA2
      AMIU=CMIU
      IF (TEMP(1).LE.90.) GO TO 10
      AMIU=CMIU-(CMIU-HMIU)*(TEMP(1)-90.0)/DFTM
      IF (AMIU.LE.0.01) WRITE (6,20) AMIU
10    ACOF=PRSA*WDTH*AMIU*RI*RI*(COS(TETA1)-COS(TETA2))/(RTIER*SIN(TETAA
     1))
      DCCE=NSHU*G*ACOF/W
      ECEN=RD*RI
      C=ECEN*SIN(TETA2)/COS(TETA2/2)
      BFR=AMIU*PRSA*WDTH*RI*RI/SIN(TETAA)
      BNR=PRSA*WDTH*RI*ECEN/SIN(TETAA)
      T=BFR*(COS(TETA1)-COS(TETA2))
      SRFC=(TETA2-TETA1)*RI*WDTH
C
      RO=RI+THK
      VOL=PI*(RO*RO-RI*RI)*WDTH
C   FRICTION MOMENT
C
      FRMNT=BFR*(COS(TETA1)-COS(TETA2)-(ECEN/(2.*RI))*((SIN(TETA1))**2-(
     1SIN(TETA2))**2))
C
C   MOMENT OF THE NORMAL FORCE
C
      ANRMNT=BNR*((TETA2-TETA1)/2.-0.25*(SIN(TETA2*2.)-SIN(TETA1*2.0)))
C
C   ACTUATING FORCE
C
      ACFRC=(ANRMNT-FRMNT)/C
      RETURN
C
20    FORMAT (10X,35HTHE FRICTION COEFF.IS TOO SMALL   =,E12.5)
      END
```

```
C                    SUBROUTINE OUTPUT
C
C
C  THIS SUBROUTINE WRITES THE RESULTS
C
C
      SUBROUTINE OUTPUT (ICALC)
      DIMENSION TEMP(50),TEPL(50),A(50),RES(50),TC(50),NVT(20),VT(20,4)
      COMMON /GLOBCM/ RI,T,WDTH,TMAX,TETA1,TETA2,FRMNT,ANRMNT,ACFRC,C,Q,
     1RD,CMIU,HMIU,AMIU,SRFC,RO,THK,DX,RTIER,W,DCCE,TOT,ECEN,NWRT,TIME,V
     2OL,TEPL,NWR,NWRA,NWRQ,NEL,NSEG,PI,G,K,HCNV,SPHT,RHO,DTAU,DFTM,TEMP
     3,RES,TC,BIO,FUR,NVT,VT,NELO,NELT,TETAA,ACOF,TINI,ZMAN,NSHU,PRSA
      WRITE (6,10) TETA1,TETA2,TETAA,PRSA,WDTH
      WRITE (6,20) RI,THK
      WRITE (6,30) K,HCNV,SPHT,DFTM,CMIU,HMIU,TINI,RHO,W,RTIER,DTAU
      WRITE (6,40) FRMNT,ANRMNT,ACFRC,C,ECEN,RD,AMIU,T
      WRITE (6,60)
      WRITE (6,50) TOT,TEPL(1),TEPL(NELO)
      WRITE(6,70) TMAX
70    FORMAT (/5X,'MAXIMUM INSIDE DRUM TEMP.=',E12.5)
      RETURN
C
10    FORMAT (//,5X,10HTETA1     =,E12.5/5X,10HTETA2     =,E12.5/5X,10HTETA
     1A A    =,E12.5/5X,10HPRESSUREA=,E12.5/5X,10HWIDTH     =,E12.5)
20    FORMAT (5X,15HINSIDE RADIUS =,E12.5/5X,15HDRUM THICKNESS=,E12.5)
30    FORMAT (//,5X,20HCONDUCTIVITY COEFF.=,E12.5/5X,20HCONVECTION COEFF
     1.  =,E12.5/5X,20HSPEC. HEAT COEFF.  =,E12.5/5X,20HMAX.TEMP.DIFFERE
     2NCE=,E12.5/5X,20HCOLD FRICTION COEFF=,E12.5/5X,20HHOT FRICTION COE
     3FF.=,E12.5/5X,20HINITIAL TEMPERATURE=,E12.5/5X,20HDRUM DENSITY
     4   =,E12.5/5X,20HCAR WEIGHT          =,E12.5/5X,20HWHEEL RADIUS
     5   =,E12.5/5X,20HTIME STEP           =,E12.5)
40    FORMAT (//,5X,19HFRICTION MOMENT    =,E12.5/5X,19HNORMAL MOMENT
     1 =,E12.5/5X,19HACTUATING FORCE    =,E12.5/5X,19HDIST. FORCE-PIVOT =
     2,E12.5/5X,19HDIST. CENTER-PIVOT=,E12.5/5X,19HRATIO A/R          =,E
     312.5/5X,10HMIU      =,E12.5/5X,10HTORQUE    =,E12.5)
50    FORMAT (/5X,F7.2,2X,2E15.4)
60    FORMAT (/5X,41HTOT.TIME      INSIDE TEMP.     OUTSIDE TEMP.)
      END
```

APPENDIX E

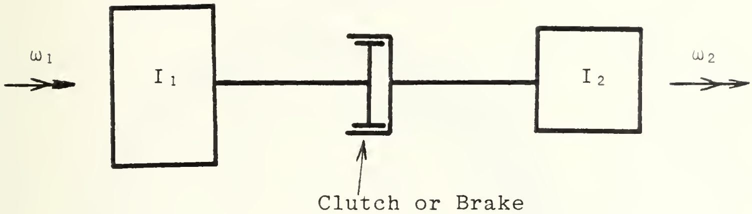

Fig. 1 Dynamic Representation of a Brake or Clutch

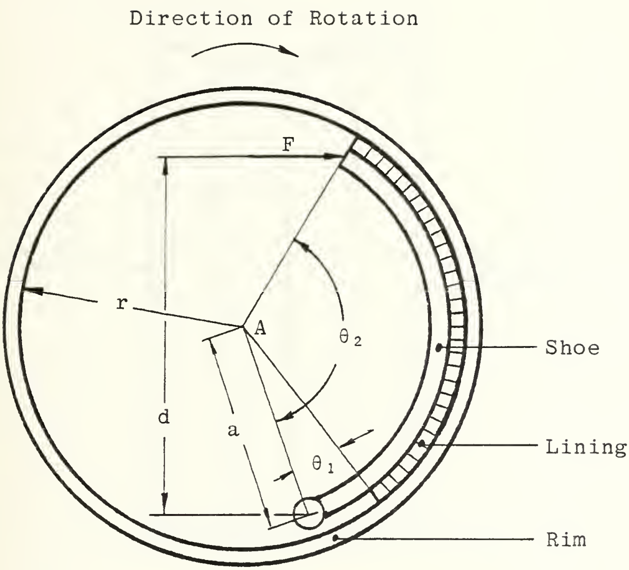

Fig. 2 Brake Assembly

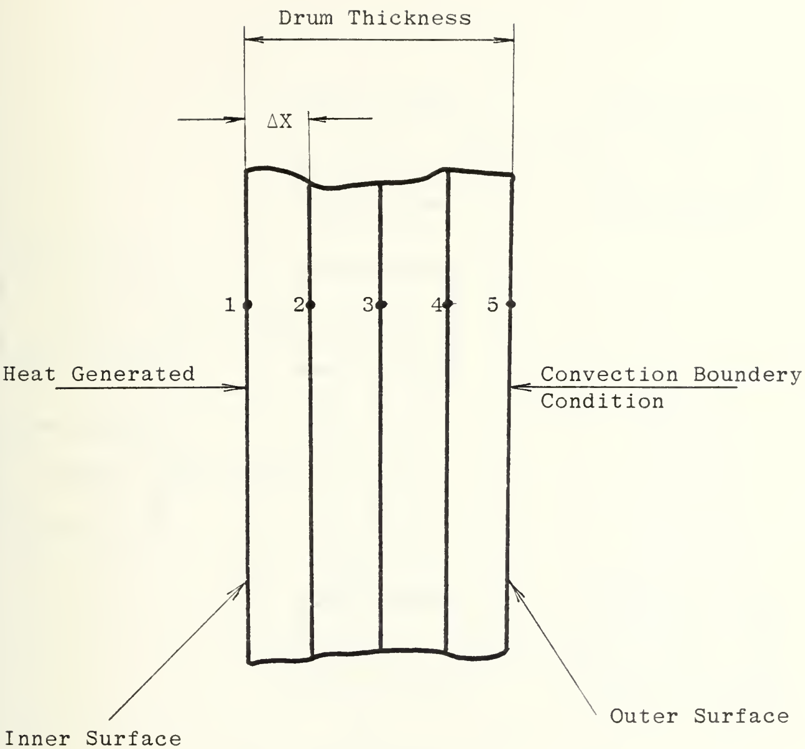

Fig. 3 Finite Difference Model

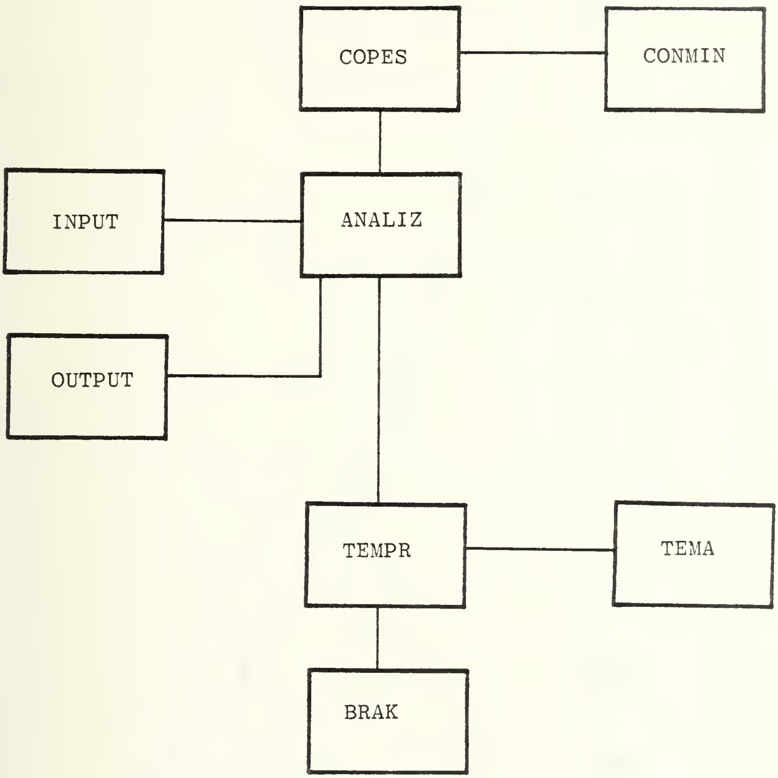

Fig. 4 Block Diagram of the Program

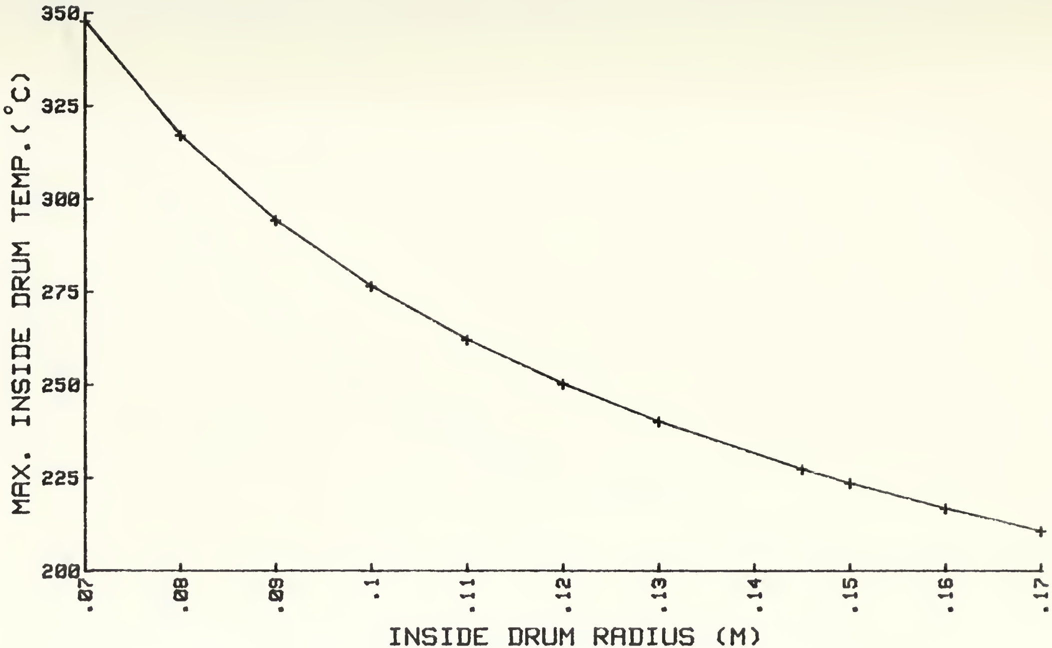

Fig. 5 Maximum Inside Drum Temp. Vs. Inside Drum Radius

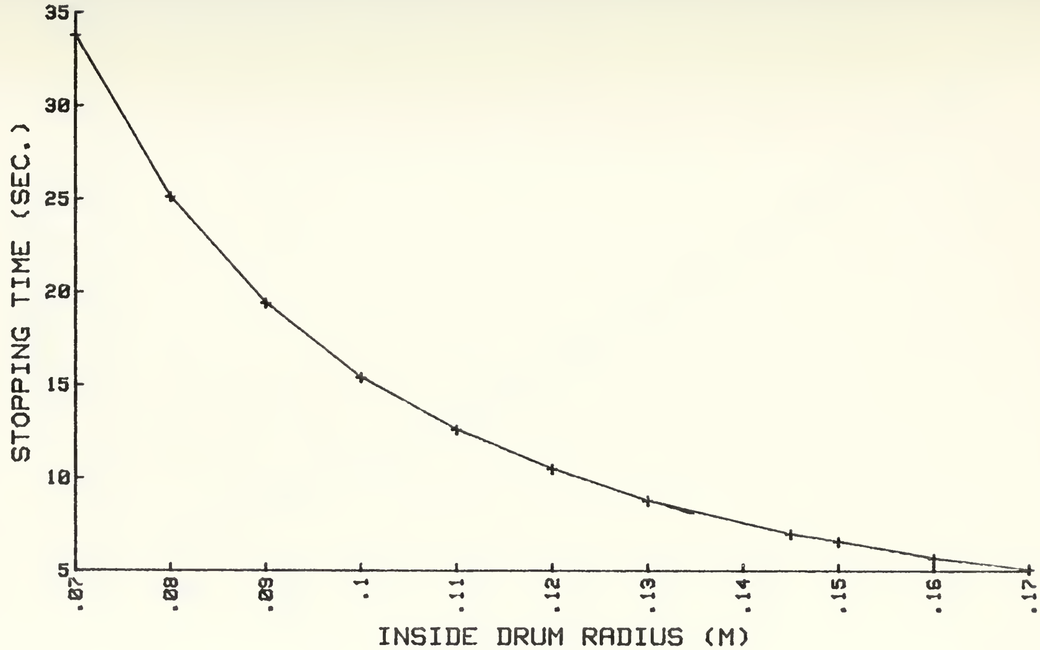

Fig. 6 Stopping Time Vs. Inside Drum Radius

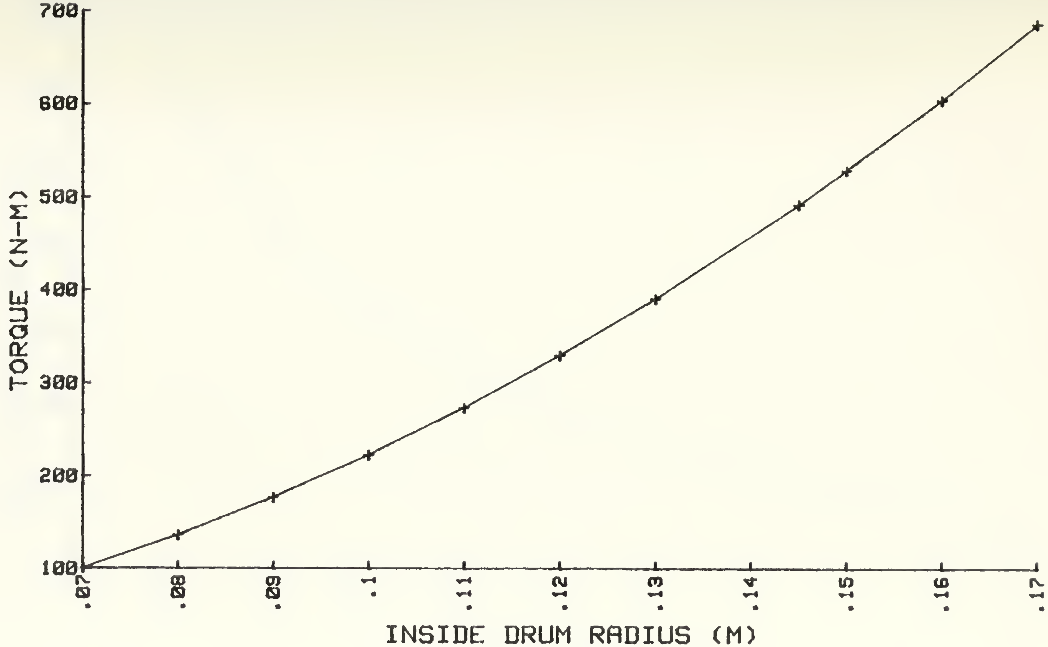

Fig. 7 Torque Vs. Inside Drum Radius

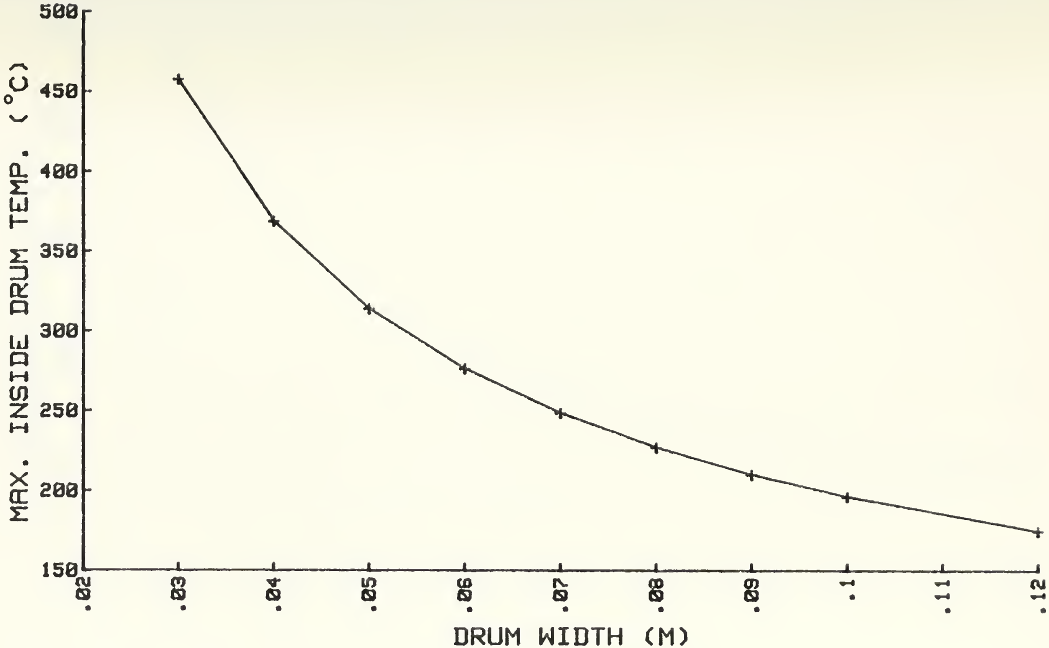

Fig. 8 Maximum Inside Drum Temp. Vs. Drum Width

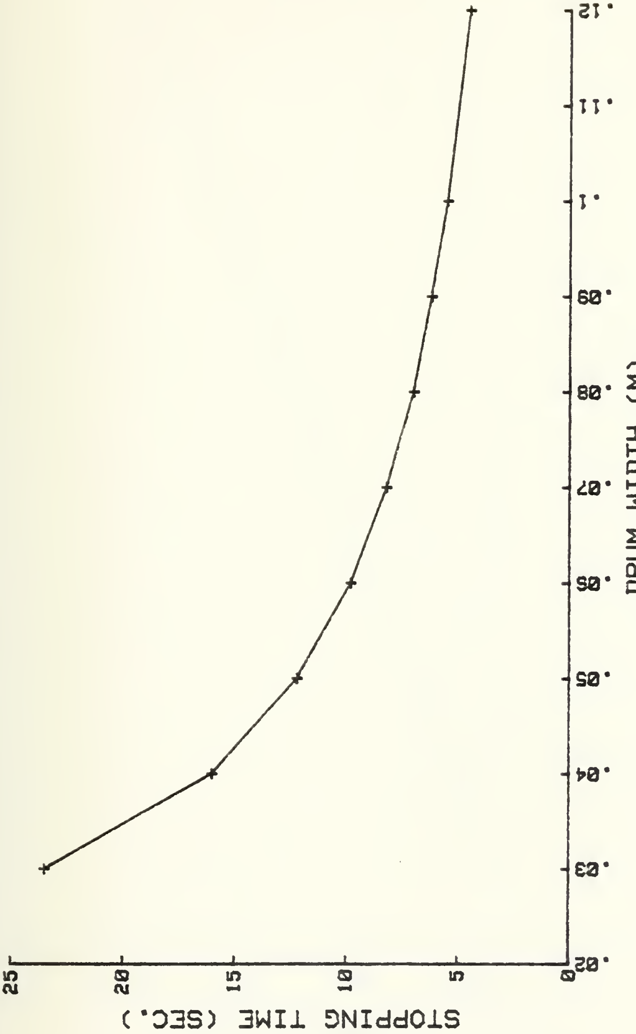

Fig. 9 Stopping Time Vs. Drum Width

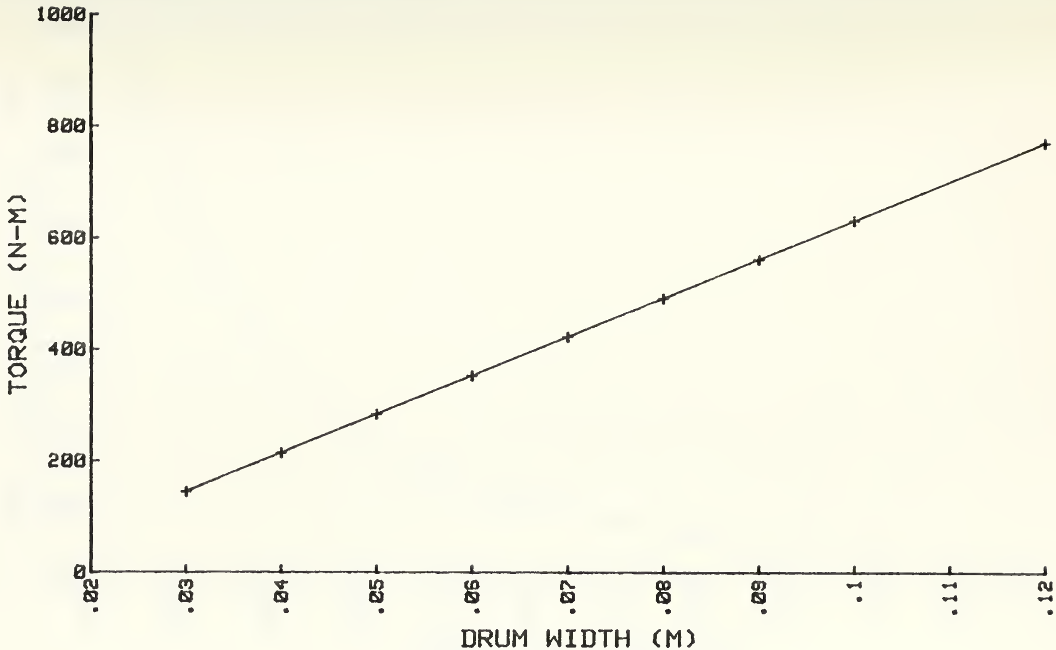

Fig. 10 Torque Vs. Drum Width

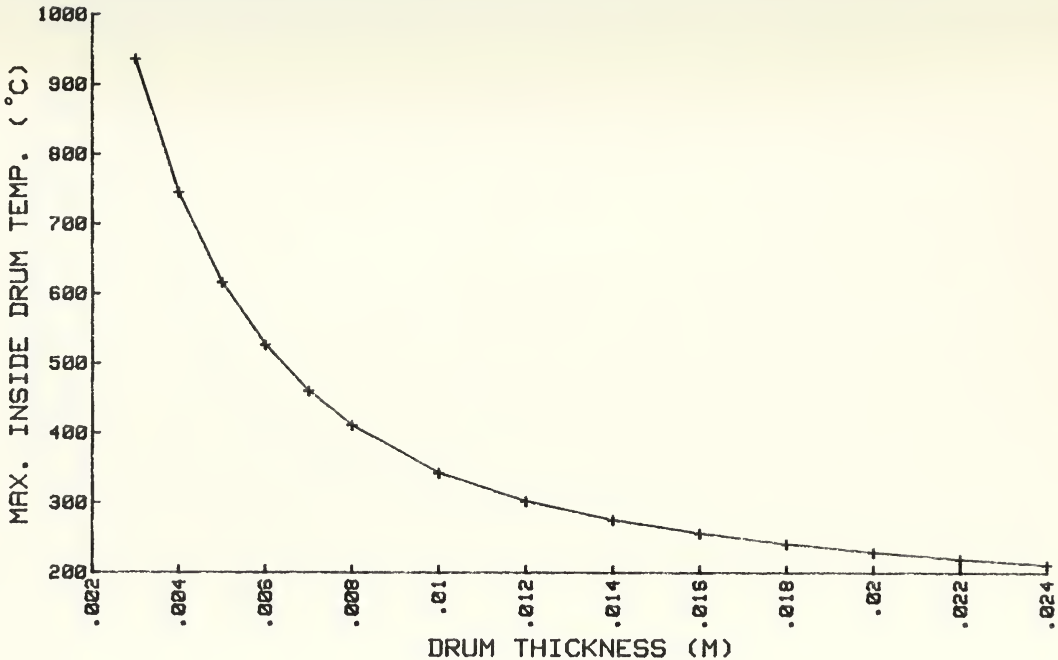

Fig. 11 Maximum Inside Drum Temp. Vs. Drum Thickness

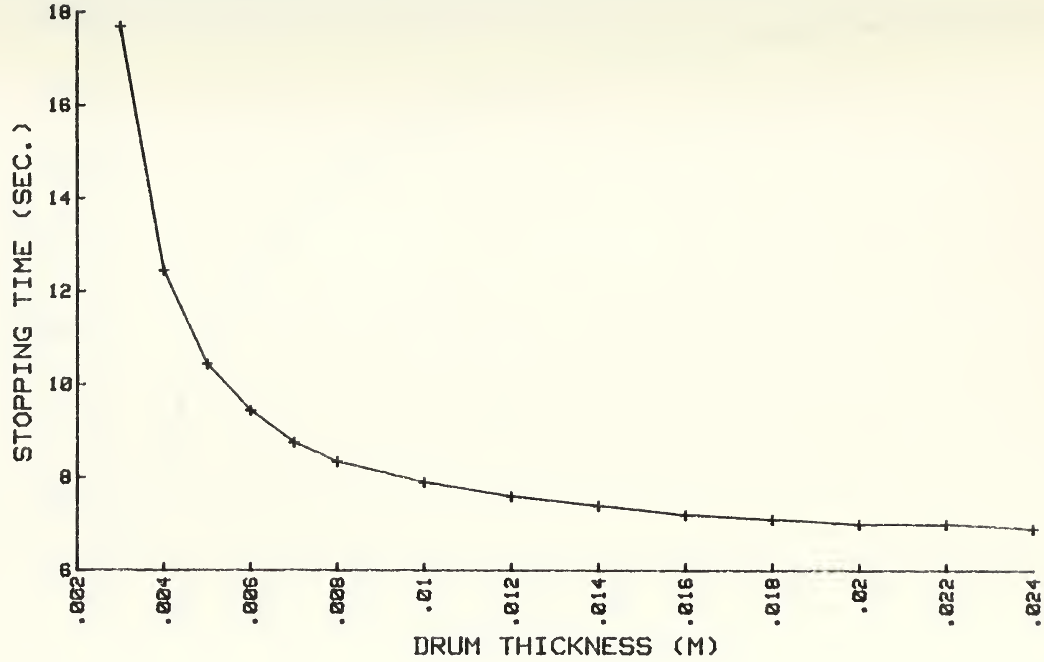

Fig. 12 Stopping Time Vs. Drum Thickness

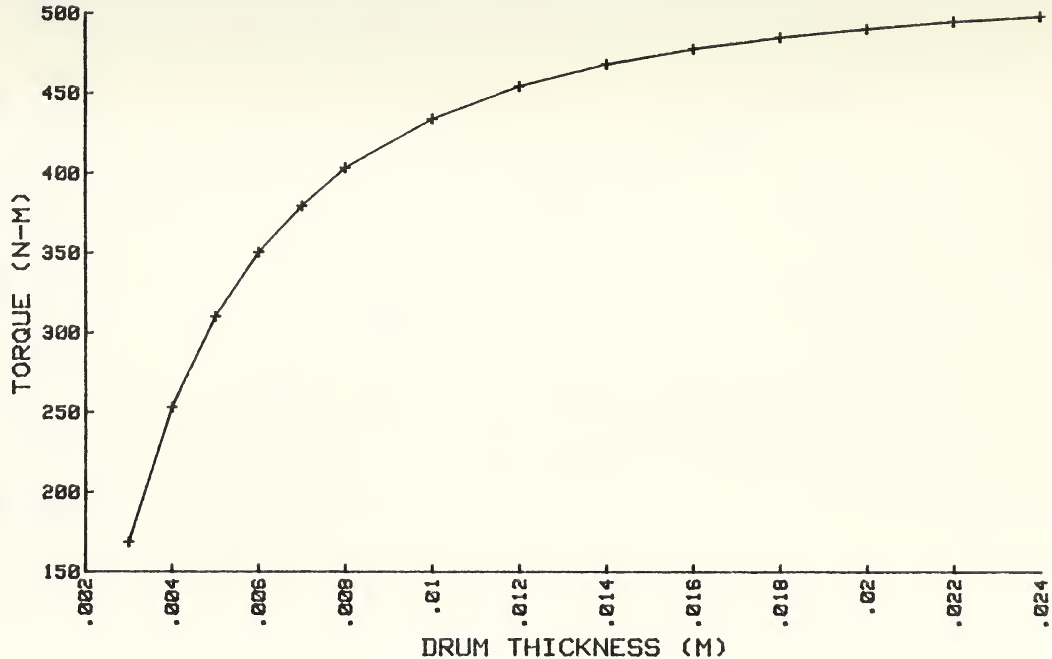

Fig. 13 Torque Vs. Drum Thickness

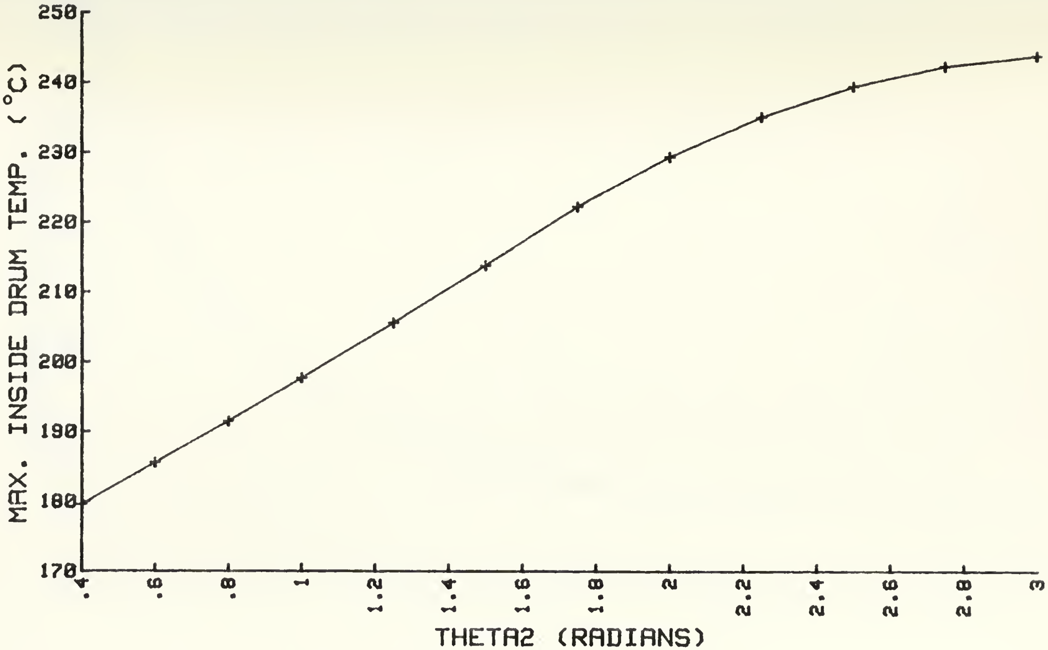

Fig. 14 Maximum Inside Drum Temp. Vs. Theta 2

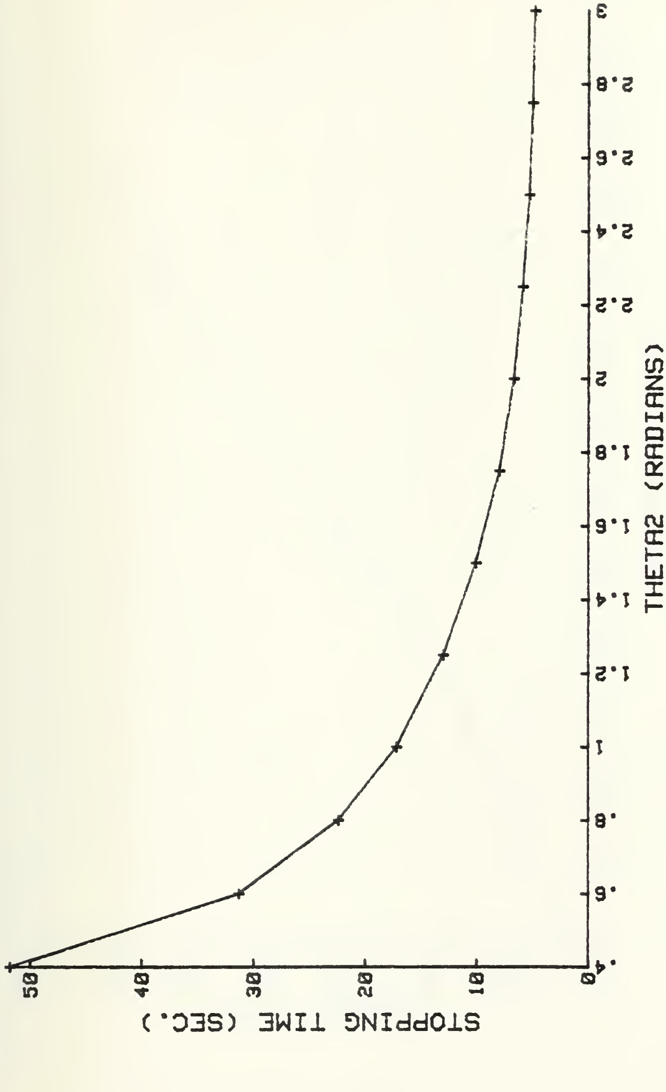

Fig. 15 Stopping Time Vs. Theta 2

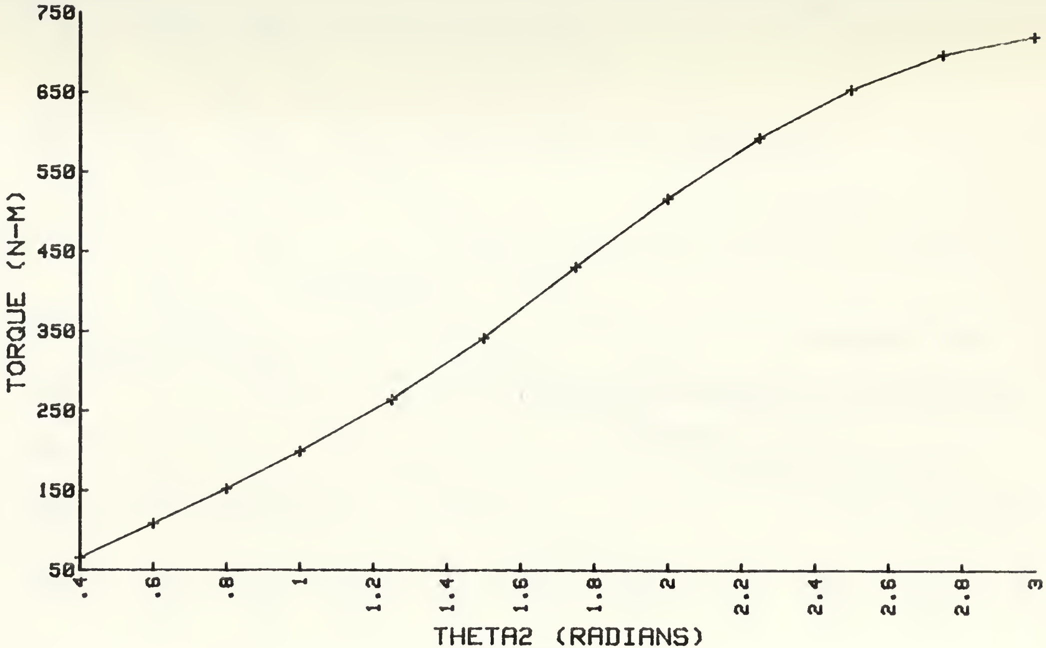

Fig. 16 Torque Vs. Theta 2

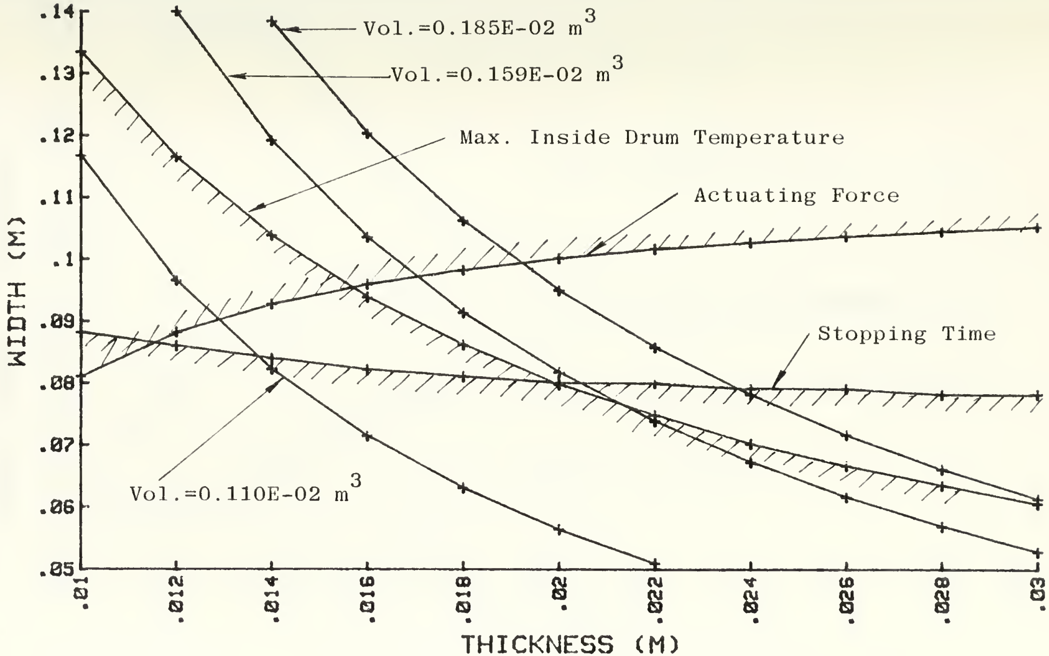

Fig. 17 Two Variable Function Space

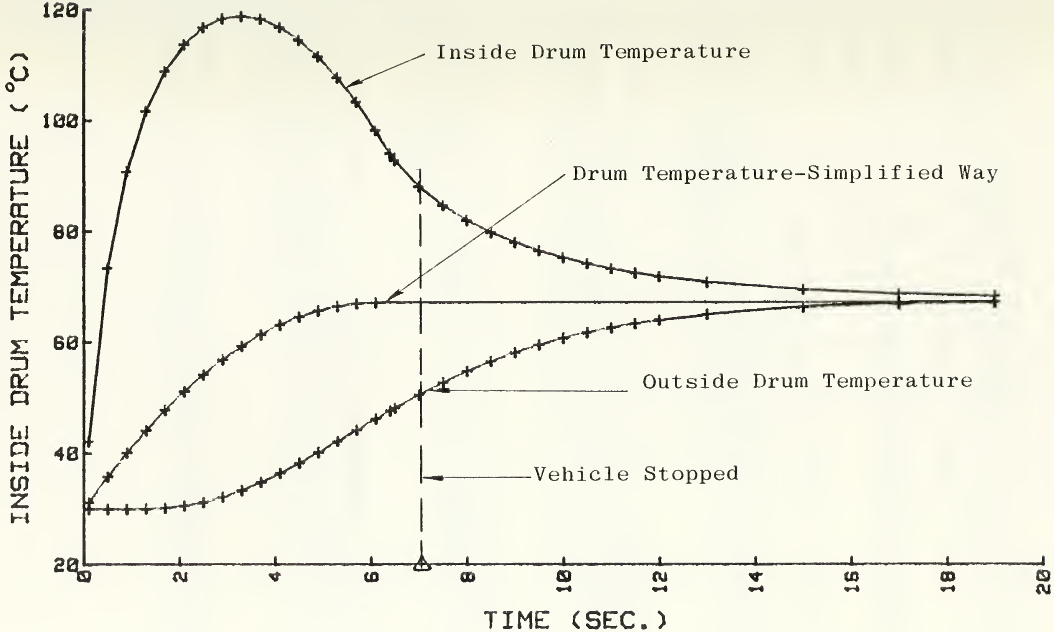

Fig. 18 Drum Temperature Vs. Time-Comparison

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